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# Stress Concentration and its Mitigation Techniques in Flat Plate with Singularities—A Critical Review

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**Abstract.** A number of analytical, numerical and experimental techniques are available for reduction of stress concentration factor around discontinuities. Determination of stress concentration factor around different discontinuities in a rectangular plate made up of different materials under different loading conditions has been reported in literature. Mitigation of stress concentration factor although, an area with very earlier findings however, is lesser reported in the literature. An attempt has been made in the present work to present an overview of various techniques developed for analysis and mitigation of stress concentration factor. The methods compared are tabulated with their findings. No single method can be suggested for each and every condition to the reduction of stress concentration; still the present comparative study will help in deciding the stress mitigation method for a particular case. Singularities of circular hole in rectangular plate and elliptical hole in rectangular plate are considered in present study. Separate nomenclature has not been given and the terms are defined as and where used. References are used as per their occurrence and not in a particular order.

**Keywords:** Stress concentration factor, discontinuity, mitigation.

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# 1. Introduction

In recent years, researchers have put enormous amount of effort in investigating techniques for analysis and mitigation of stress concentration. The failure of structures due to stress concentration at any discontinuity has been baffling engineers for long. It has been found that structure failures in ships, offshore structures, boilers or high rise buildings subjected to natural calamities is due to stress concentration. Stress concentration mainly occurs due to discontinuities in continuum. Due to stress concentration the magnitude of the maximum stress occurring in any discontinuity is comparatively higher than the nominal stress. Stress concentration cause strength degradation and premature failure of structures because of fatigue cracking and plastic deformation frequently occurring at these points.

## 1.1. Stress Concentration

Stress concentration is localization of high stresses mainly due to discontinuities in continuum, abrupt changes in cross section and due to contact stresses. To study the effect of stress concentration and magnitude of localized stresses, a dimensionless factor called Stress Concentration Factor (SCF), $K_t$ as defined by Eq. (1) is used.

$$K_t = \sigma_{max} / \sigma_{nom} \tag{1}$$

where $\sigma_{max}$ is maximum stress at the discontinuity and $\sigma_{nom}$ is nominal or background stress.

A rectangular isotropic or orthotropic plate with circular hole under different types of loading have found wide application in various fields of engineering like in automobiles, marine, aerospace and in other engineering structures. In these plates SCF is affected by many parameters viz. plate length, diameter of holes, dimensions of discontinuity, thickness of plate, elastic constants and many more. For design of such plates with holes/singularities, knowledge of deflection, stresses and stress concentration are required.

The stress concentration factor can be determined analytically by applying elasticity theory. For a large thin plate with a small circular hole at the center, that is subjected to uni-axial far-field tension, $\sigma$, acting along the x-axis, the stresses (radial, circumferential and tangential ) around the vicinity of the hole are given in polar coordinates $(r,\theta)$ by following equations [1], Fig. 1. :

$$\sigma_{rr} = \frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)\left(1-\frac{3a^2}{r^2}\right)\cos 2\theta \tag{2}$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta \tag{3}$$

$$\sigma_{r\theta} = -\frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1+\frac{3a^2}{r^2}\right)\sin 2\theta \tag{4}$$

where $\theta$ = Angle measured counter clockwise from x-axis and $a$ = Radius of the circular hole.

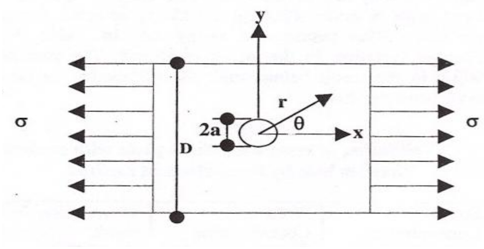


Fig. 1. Plate with single circular hole subjected to uni-axial stress.

## 1.2. Measurement of Stress

Stress analysis of the critical elements under various loading conditions is carried out by the researchers for safe design of the element. Stress is measured by experimental methods or analytical/numerical method.

### 1.2.1. Experimental Methods

From the various experimental methods available for stress analysis, the following methods are long established.

#### 1.2.1.1. Photo Elasticity

Photo elastic stress analysis is a full field technique for measuring the magnitude and direction of principal stresses. When polarised light is passed through a stressed transparent model, interference patterns or fringes are formed. These patterns provide immediate qualitative information about the general distribution of stress, positions of stress concentrations and of areas of low stress using the principals of stress optic law, Eq. (5) [2].

$$\sigma_1 - \sigma_2 = N\, f_\sigma / t \tag{5}$$

where $\sigma_1$ and $\sigma_2$ are the values of the maximum and minimum principal stresses at the point under consideration, N is the fringe number or fringe order at the point, $f_\sigma$ is the material fringe value and t is the model thickness.

#### 1.2.1.2. Brittle Coating

The brittle-lacquer technique of experimental stress analysis relies on the failure by cracking of a layer of a brittle coating which has been applied to the surface under investigation. Specially prepared lacquers are usually applied by spraying on the actual part. Pattern of small cracks appear on the surface of this coating where the strain is high indicating the presence of stress concentration. The cracks also indicate the directions of maximum strain at these points since they are always aligned at right angles to the direction of the maximum principal tensile strain. These crack data could be used to locate strain gauges for precise measurement of the stress. The method is however, sensitive to temperature and humidity [2].

#### 1.2.1.3. Electrical Strain Gauges

The method is one of the most popular and widely accepted for strain measurements and stress analysis. The strain gauge consists of a grid of strain-sensitive metal foil bonded to a plastic backing material. Any change in length will result in a change of resistance. Thus measurement of this resistance change with suitably calibrated equipment enables a direct reading of linear strain to be obtained.

Change of resistance and strain may be expressed as follows:

$$\Delta R / R = K \times \Delta L / L$$

$$K = (\Delta R / R) / (\Delta L / L)$$

$$\varepsilon = (\Delta R / R) / K \tag{6}$$

where $\Delta R$ and $\Delta L$ are the changes in resistance and length respectively, K is termed as the gauge factor and $\varepsilon$ is the strain. Thus measurement of this resistance change with suitably calibrated equipment enables a direct reading of linear strain to be obtained.

### 1.2.2. Analytical/Numerical Methods

Distribution of stresses in a structure with boundary conditions, i.e. displacements and/or forces on the boundary can be determined by using either the closed form analytical methods or by approximate numerical methods.

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Boundary value problems can be solved analytically by using constitutive equations based on the elastic or plastic behavior of the material under load. Analytical or close-form solutions can be obtained for simple geometries, constitutive relations and boundary conditions.

Approximate solutions for boundary-value problems can be obtained through the use of numerical methods such as finite element method, finite difference method, boundary element method, finite volume method and mesh less method.

#### 1.2.2.1. Finite Element Method

The structural model to be analysed is divided into many small pieces of simple shapes called elements. Finite Element Analysis (FEA) program writes the equations governing the behaviour of each element taking into consideration its connectivity to other elements through nodes. These equations relate the unknowns, for example displacements in stress analysis, to known material properties, restraints and loads. The program assembles the equations into a large set of simultaneous algebraic equations - thousands or even millions. These equations are then solved by the program to obtain the stress distribution for the entire model.

In recent years, with the advent of advanced software's, the FEA based software ANSYS, COSMOL, DIANA, ABACUS and NASTRAN have been very useful for stress analysis. These software's are preferred by users according to the type of stress analysis, the type of elements to be analysed and the depth of accuracy required.

#### 1.2.2.2. Boundary Element Method

In this method the governing differential equation is converted into an integral form, often involving only integrals over the boundary of the domain. Consequently, only the boundary has to be discretized in order to carry out the integrations. The dimensionality of the problem is thereby effectively reduced by one. A three dimensional volume problem becomes a two dimensional surface one, while a two-dimensional plane problem involves only one-dimensional line integrations. Also, because the interior of a solution domain is not discretized, there is much less approximation involved in representing the solution variables and rapid variations of, for example, stresses and displacements can be resolved very accurately. Stresses are accurate as there are no approximations imposed on the solution in interior domain points. The method is suitable for modelling problems of rapidly changing stresses. Boundary Element Method (BEM) uses less number of nodes and elements for the same level of accuracy as other methods. The Boundary Element Method is unsuitable if information is required at a large number of internal points [3].

#### 1.2.2.3. Mesh Free Method

It is an alternative numerical approach to eliminate the well-known draw backs in the finite element and boundary element methods. Mesh free method is especially useful in the problems with discontinuous or moving boundaries. The main objective of the Mesh Free methods is to get rid of the difficulty of meshing and re-meshing the entire structure, by only adding or deleting nodes in the entire structure, instead. Mesh Free methods use a set of nodes scattered within the problem domain as well as sets of nodes scattered on the boundaries of the domain to represent the problem domain and its boundaries. These sets of scattered nodes are called field nodes.

## 2. Literature Review

Analysis of stress concentration around discontinuities in plates under various loading conditions is worked out by various researchers from as early as in 1960. Researchers have worked with different types of discontinuities like circular hole or elliptical hole in a rectangular plate, different types of notches like semi circular notch, V notch, U notch in a plate with different loading and material. Lately people have also contributed significantly in the area of stress mitigation techniques. The analysis is carried out by one of the method discussed earlier – experimental or analytical. An attempt is made to review few of the important contribution for analysis of stress concentration and its mitigation in the present work.

### 2.1. Analysis of Stress Concentration

The work carried out by various researchers for analysis of SCF is compiled and presented by Peterson [4].Circular hole in an infinite plate was analyzed by Kirsch [5] and SCF value under tensile loading is reported as 3. SCF values for finite width plate with a circular hole are determined by Howland [6] and are presented for various a/w ratios in the form of curves. The above has been formulated by Heywood [7] and the relation is as follows:

$$K_{tn} = 2 + (1 - a/w)^3 \tag{7}$$

where, $K_{tn}$ is defined as ratio of maximum stress to nominal stress. As reported by Peterson, the above formula is satisfactory for a/w <0.3.

Fedorov [8] studied the effect of D/A ratio and different loading on stress concentration in a glass reinforcement plastic specimen and extended the work on anisotropy of material. He has presented studies of 12 different models for different loads and sizes.

Stress concentration around irregular holes using complex variable methods are reported by K. R. Y. Simha [9].Conformal mapping method has been used for evaluation of stresses. The method is an operation in complex mathematics which maps a set of points in one coordinate system to a corresponding set in another, keeping the angle of intersection between two curves constant, and is widely used in solving elasticity problem. Nine hole shapes with same area and different perimeter are studied. Irregular holes may change their shape if not their size by exchanging surface energy with strain energy. A linear elastic analysis followed in this paper may not support the physics of change in shapes but can be extended to a linear visco-elastic material.

Solution of plate with an elliptical hole was first given by Kolosoff [10]. Ukadgaonkar [11] has analyzed the stresses in an infinite plate with elliptical hole or crack with tensile stresses. The closed form solutions are given for SCF and SIF (Stress Intensity Factor). It is observed that SCF and SIF depend on the material parameters for anisotropic material, while it is independent of material properties in case of isotropic materials.

N. Troyani [12], showed that theoretical SCF also depends on the length of the member in addition to the established other geometric parameters. He compared the value of SCF for short plate and long plate for same loading and boundary conditions and introduced transition length for short members. Transition length is defined as the length of the bar for which the calculated theoretical SCF varies 1% ± 0.05 % with respect to corresponding established values for long bars.

Engels [13] proposed optimal design of hole reinforcement for composite structures. Reinforcement by elliptical doublers as well as doublers is adopted. An appropriate optimization structural model is developed to describe the mechanical behaviour like displacement and stresses of such structures and an appropriate mathematical optimization algorithm is used to approach the desired optimum design. The infinite composite plate can be analyzed by the complex potential method as well as the Finite Element Method. The implemented procedure works with good reliability, efficiency and yields optimal reinforced designs which are useful for direct engineering applications.

Zirka *et al.* [14] have analyzed stress concentration around circular hole in a rectangular plate for orthotropic and isotropic plates under dynamic and static loading. They have used photo elastic method for analysis. The plate model is made of an anisotropic optically sensitive material with the following properties, Elastic modulus: $E_1 = 4.30 \times 10^3$ MPa, $E_2 = 6.08 \times 10^3$ MPa; Shear modulus: $G = 1.79 \times 10^3$ MPa; Poisson's ratios: $\gamma_{12} = 0.33$ and $\gamma_{21} = 0.471$ and optical constants: $\sigma_{d1}$=2.5Mpa.cm / fringe, $\sigma_{d2}$=4.1Mpa .cm / fringe, and $\varepsilon_d = 4.95 \times 10^{-4}$ cm/fringe. The dynamic photo-elastic method was used for dynamic loading, P(t). The stress fringe patterns of the hole were recorded at a frame rate of f = 2MHz.

The curves plotted for Dynamic SCF, $K\sigma$ - Dynamic SCF is the SCF under the dynamic loading, is shown in Fig. 2. Dynamic SCF values are compared with the theoretical values.

The experimental values of $K\sigma$ at some peripheral points of a hole ($0 \leq \theta \leq \pi$) in orthotropic and isotropic plates under dynamic and static loading is presented in Fig. 2. and are tabulated in Table 1. $(K\sigma)_{d.o.}$, $(K\sigma)_{d.i.}$, $(K\sigma)_{s.o}$ and $(K\sigma)_{s.i.}$ are SCF for dynamic loading and static loading for orthotropic and

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isotropic materials respectively. $(K\sigma)_{t.o.}$ and $(K\sigma)_{t.i.}$ presents theoretical values for an orthotropic plate (t,o) and for an isotropic plate (t,i) respectively.

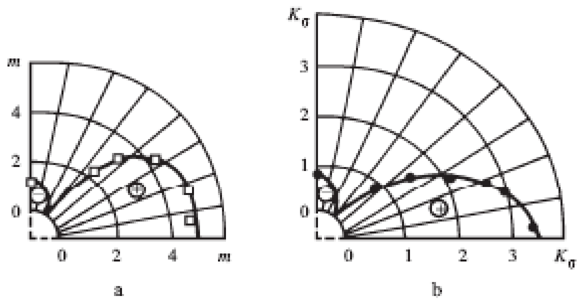


Fig. 2. Distribution of fringe order "m"; (a) and dynamic stress concentration factor "Kσ"; (b) along the periphery of a circular hole in an orthotropic plate [14].

Table 1. Experimental values of Kσ at peripheral points of a hole in an orthotropic and isotropic plate [14].

| $\theta^0$ | 0 | π/12 | π/6 | π/4 | π/3 | 5π/6 | π/2 |
|---|---|---|---|---|---|---|---|
| $(K_\sigma)$d.o. | 3.75 | 3.05 | 1.61 | 0.57 | 0.05 | -0.65 | -0.92 |
| $(K_\sigma)$d.i. | 2.72 | 2.12 | 1.16 | 0.32 | 0.04 | -0.47 | -0.66 |
| $(K_\sigma)$s.o. | 3.59 | | 1.66 | | 0.04 | | -0.89 |
| $(K_\sigma)$s.i. | 2.67 | 2.08 | 1.23 | 0.34 | 0.03 | -0.46 | -0.65 |
| $(K_\sigma)$t.o. | 3.62 | 2.85 | 1.64 | 0.56 | 0.04 | -0.62 | -0.89 |
| $(K_\sigma)$t.i. | 2.70 | 2.10 | 1.24 | 0.34 | 0.03 | -0.45 | -0.65 |

Zheng Yang *et al.* [15] Investigated the elastic stress and strain fields of finite thickness large plate containing a hole using 3D finite element method for analysis. The maximum stress and strain are taken as function of thickness of plate and Poisson's ratio of the plate. The results are given in different curve forms, for different geometrical element and Poisson's ratio. Relation between stress concentration factor and strain concentration factor - strain concentration factor is the ratio of strains in direction of loading to net strain in the element at discontinuity, has also been studied.

Mittal and Jain [16] proposed finite element analysis for determination of Stress Concentration and deflection in isotropic, orthotropic and laminated composite plates with central circular hole under transverse static loading. Plate with uniformly distributed loading and with loading at boundary of hole was considered for analysis, as shown in Fig. 3. The effect of D/A ratio upon SCF for normal stress in X and Y directions is reported. Variation in SCF for different boundary conditions at the edges of plate has been analyzed.

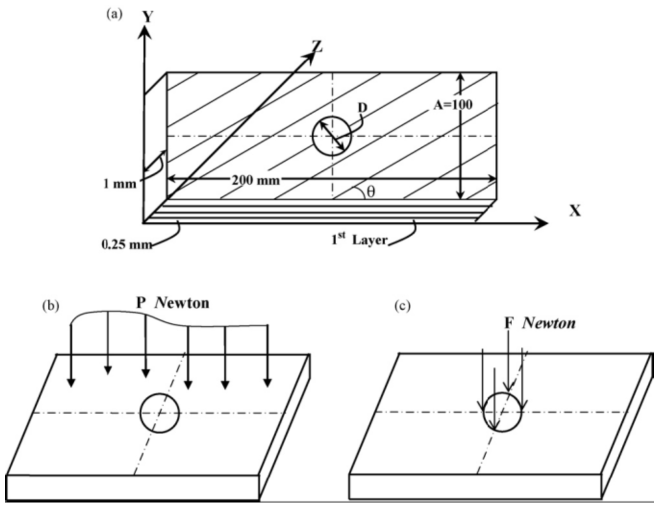


Fig. 3. Plate with circular hole under different loading conditions: (a) A laminated plate with central hole; (b) Uniformly distributed loading of *P* Newton; (c) Loading at boundary of hole. [16]

The equations have been developed by Roark to fit as closely as possible the many data points given in the literature by many researchers through experiments and analytical methods. Roark developed the following equation for SCF based on the work of Flynn and Heywood [7]:

$$Kt=3.0 - 3.13(2r/D) + 3.66(2r/D)^2 -1.53(2r/D)^3 \quad (8)$$

where 2r = Diameter of hole and D = Width of plate .

T. Hasan [17] has reported stress analysis of steel plate having holes of various shapes, sizes and orientations using finite element method. Finite element analysis is carried out by using the commercial software COMSOL 3.3. The rectangular plate with elliptical hole subjected to uni-axial load used for analysis is shown in Fig.4.

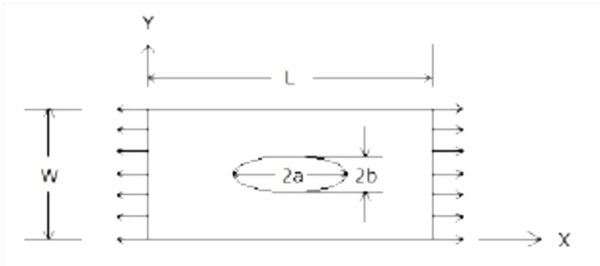


Fig. 4. Rectangular plate with elliptical hole subjected to uni-axial load [17].

Results are presented in the form of graphs as shown in Fig. 5(a). Effects of hole shape are critically analyzed from the results of the finite element method and analytical methods. Comparisons between the results by the present finite element method and the analytical solution technique yields good agreement.

For elliptical hole, with the increase of plate length to width ratio the maximum stress at all angular position increases as shown in Fig. 5(a). At 0° angular position of elliptical hole, maximum stress occurs at the two ends of hole on its minor axis. At 90° angular position of elliptical hole, maximum stress occurs at the two ends of hole on its major axis Fig. 5(b).

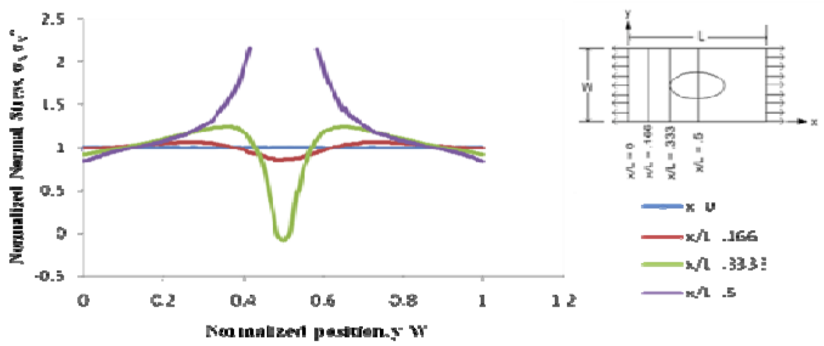


Fig. 5(a). Distribution of normalized axial stress component at different sections of the plate [17].

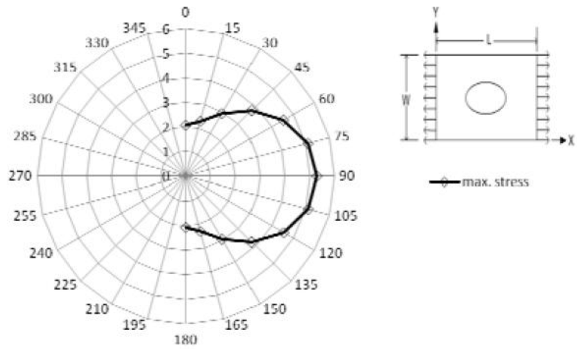


Fig. 5(b). Variation of maximum stress due to the orientation of elliptical hole [17].

Several persons have compiled the work on analysis of SCF of various researchers. The work of Peterson and Roark is commendable in this direction. However with the advent of new technology and new materials the latest work in this area need to be compiled. The review presented above is a small effort in this direction and is further tabulated below.

Mohsen [18] first discussed the effect of non-homogeneous stiffness on SCF by keeping Poisson's ratio constant. Secondly, Young's modulus is held constant and influence of varying Poisson's ratio is analyzed. SCF is analyzed for biaxial loading as well as for pure shear.

Kubair [19] numerically investigated the effect of the material property in-homogeneity on the SCF due to a circular hole in functionally graded panels. Functionally graded materials are composites in which the material properties vary continuously as a known function of the spatial position. A parametric study was performed by varying the functional form and the direction of the material property gradation. The results from parametric study showed that the SCF is reduced when the material property (Young's modules) progressively increased away from the centre of the hole. In exponential functionally graded materials, the angular variation and the magnitude of the stresses are affected by the material property inhomogeneity.

In Table 2, contributions of work from many researchers have been summarized. Different techniques over the years have been used for analysis of SCF by the researchers. These have been compared on the basis of type of discontinuity in the element like notches or holes, circular/elliptical hole etc., the types of material used for the element, different types of loading applied to the element like axial transverse, biaxial loading etc. Further, the method used for analysis of SCF is also varies from different researchers.

Table 2. Review of stress concentration analysis in plate with singularities.

| S. No. | Author | Year | Discontinuity | Material | Loading | Analysis Techniques | Review |
|---|---|---|---|---|---|---|---|
| 1 | R. E. Peterson [4] | 1966 | Holes, notches | Isotropic | Axial , bending moment, twisting moment | Compiled work of many researchers | Useful for researchers and design engineers |
| 2 | A. P. Fedrov and D. Popov [8] | 1972 | Circular hole in plate | Glass reinforced plastic | Uni-axial, Bi-axial | Experimental,12 models were analyzed | Effect of D/W ratio, loading on Stress and strains around circular hole are studied |
| 3 | J. B. Hanus and C. P. Burger [20] | 1981 | Elliptical hole near an edge | Orthotropic | Uni-axial Tensile | Photo elasticity | Formed parameterized geometry models & studied interaction between holes & free edges |
| 4 | N. Troyani, C. Gomes, and G. Sterlacci [12] | 2002 | Short rectangular plate with central Circular hole | Orthotropic | Uniform Tension | Finite Element Method | Introduced Theoretical SCF for short plate and effect of transition length |
| 5 | A. I. Zirka [14] | 2004 | Circular Hole in a plate | Orthotropic | Static & Dynamic | Photo elastic method | Experimental & theoretical values, of dynamic SCF & static SCF are similar. Dynamic SCF can be approx. found by using static SCF & dynamic elastic characteristic of the material. |
| 6 | L. Toubal, M. Karama, and B. Lorrain [21] | 2004 | Circular hole | Composite | Longitudinal tensile load | A non-contact measurement method, electronic speckle pattern interferometer (ESPI) | Stress concentration characterization study of a Laminate has been carried out. The strain concentration is influenced by the loading direction. |
| 7 | F. Li, Y. He, C. Fan, H. Li, and H. Zhang [22] | 2007 | Two equal circular hole | Aluminium alloy LY12-CZ plate | Axial and transverse tension | 3D Finite element method | The effect of plate thickness on the SCF is studied. The relationship between SCF & the distance between holes is also analyzed. |
| 8 | N. D. Mittal and N .K. Jain [23] | 2007 | Circular hole in rectangular plate | Isotropic, Orthotropic and Composite | Transverse Static Load ,UDL | Finite element method | Analyzed stress concentration and deflection, effect of diameter of hole to width of plate has also been reported |
| 9 | T. Hasan [17] | 2009 | Plate having holes of various shapes, sizes and orientations | Orthotropic | Axial loading | Finite Element Method | Effects of hole shape on SCF are critically analyzed |
| 10 | M. Mohammadi, J. R. Dryden, and L. Jiang [18] | 2011 | Circular hole in an infinite plate | Functionally Graded Material ( FGM) | Uniform biaxial tension and pure shear | Analytical | The effect of non-homogeneous stiffness and varying Poisson's ratio upon SCF are analyzed. |

## 2.2. Mitigation of Stress Concentration

Stress concentration in the vicinity of singularity can be reduced by smoothening the stress flow lines around the singularity. Some of the approaches used for mitigation are removal of material from the vicinity by introduction of auxiliary holes, reinforcement of the hole by adding material, hole shape

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optimization etc. Researchers attempted the means of stress mitigation using experimental or analytical stress analysis methods.

Rajaiah *et al*. [24] proposed hole shape optimization for stress mitigation in a finite plate by photo elasticity method. They introduced auxiliary holes around main hole for mitigation of SCF and also optimized the shape of circular holes. Figure 6(a) represents the results through the graph. The effort is made by experimental determination of reduction in SCF by a) introduction of circular holes b) optimization of shape of main hole c) optimizing the shape of main hole as well as auxiliary holes, Fig. 6(b).

It is reported that the shape optimization of main hole and auxiliary shapes result in mitigation of SCF, however feasibility of shape optimization in all the cases should be checked before using the method.

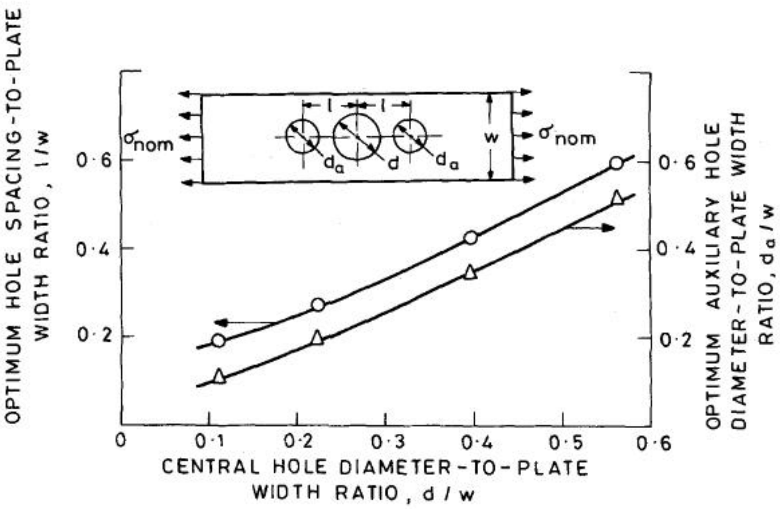


Fig. 6(a). Curves showing relation for optimum auxiliary hole size and spacing [24].

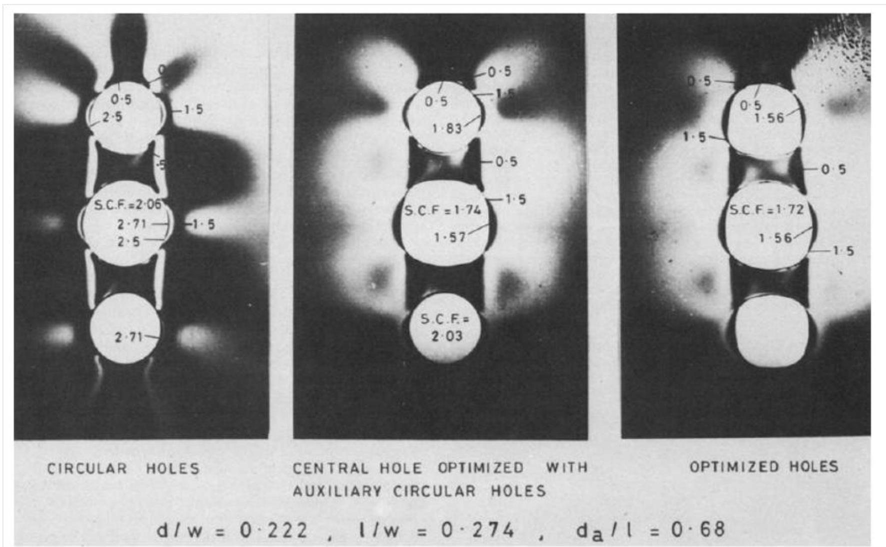


Fig. 6(b). Photo-elastic results of plate with main hole and auxiliary holes [24].

Meguid [25] presented a technique for reduction of SCF in a uni-axially loaded plate with two coaxial holes by introducing defence hole system- material removal in the form of circular holes Fig. 7. Defence hole system is a technique of material removal for stress mitigation. Finite element method was used for analysis. A comprehensive plane stress finite element study of the effect of material removal upon mitigation of elastic SCF in a uni-axially loaded plate with two co axial holes was made. Three systems for defence holes were described for FEM analysis as shown in Fig. 7. Reduction in maximum SCF ranging from 7.5% to 11 % could be achieved. They validated the results with the existing analytical solutions.

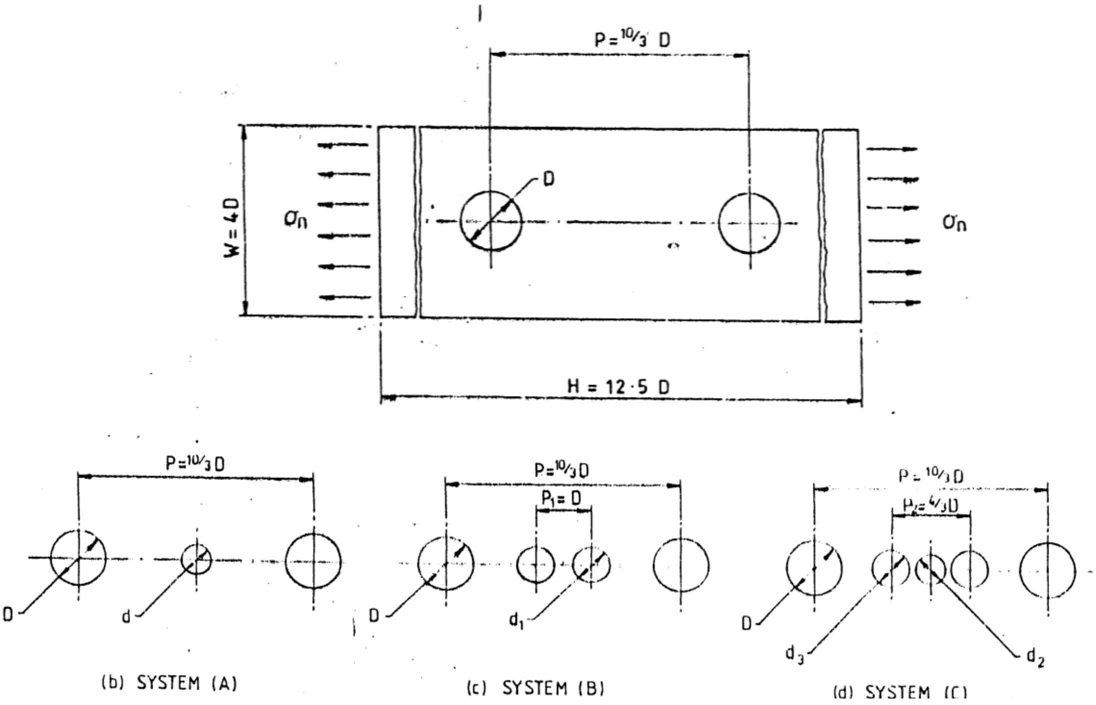


Fig. 7. Plate with two circular holes and proposed defence hole systems [25].

Giare *et al.* [26] presented a method for the reduction of stress concentration in an isotropic plate by using composite material rings around the hole. Figure 8 shows the reinforcement in the circular hole in plate. They have reported the reduction in stress concentration factor by reinforcement. The present reinforcement technique may be found very useful in composite laminate plates where they could be incorporated during manufacturing.

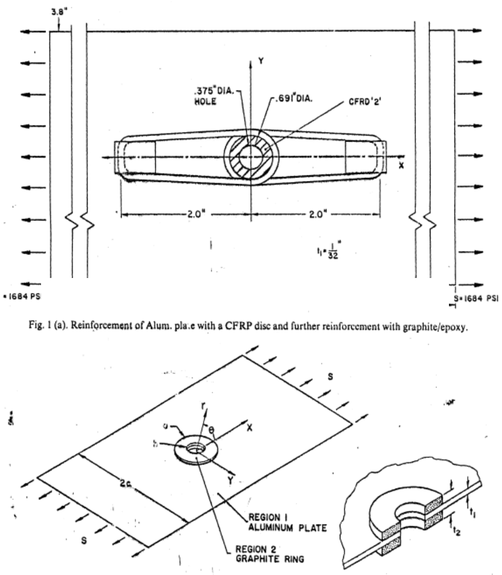


Fig. 8. Reinforcement of the circular hole in plate [26].

Sanyal and Yadav [27] have extended the work of Heywood and others by proposing the optimum distance and size of auxiliary holes for mitigation of SCF. By introducing the auxiliary holes in the line of original hole, about 17% mitigation in SCF is achieved. They have proposed an optimum distance between the original hole and relief hole and also an optimum size of relief hole by assuming elliptical stress flow lines Fig. 9.

The following expression is suggested for the optimum distance δ between the centres of holes in terms of radius of main hole $a_1$ and radius of auxiliary hole $a_2$.

$$\delta = \sqrt{3}\, a_1 + a_2 \qquad (9)$$

$$a_2 = 0.85 \;\; a_1 \qquad (10)$$

The work is also extended by introduction of multiple relief holes Sanyal and Yadav [28] using the optimum distance and size of relief holes.

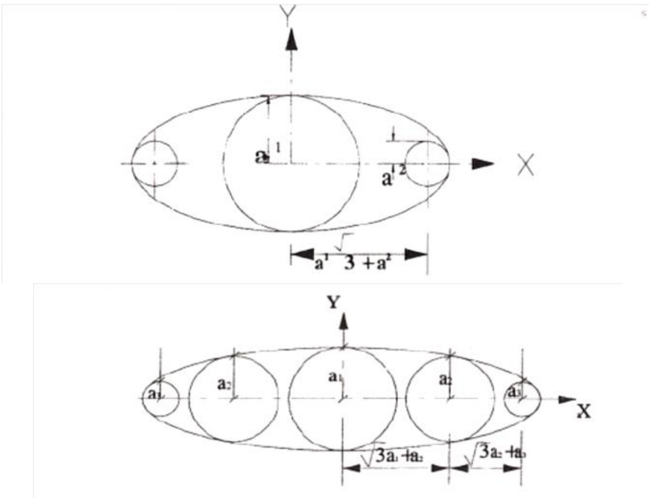

Fig. 9. Main hole with proposed relief holes [27].

Mittal and Jain [23] proposed optimization of design of square simply supported isotropic plate with central circular hole subjected to transverse static loading by Finite Element Method. They have reported around 30% reduction in SCF. They proposed four auxiliary holes around circular hole in square plate. A further modification of work was carried out by optimization of auxiliary hole shape by giving elliptical shape to the auxiliary holes.

Optimal hole shape for minimum stress concentration in two dimensional finite plates is given by Zhixue Wu [29]. They extended the optimality criteria of uniform energy density to the problem of two dimensional shape optimization with curvature constraints. The solutions of fillets and holes in plates show that the proposed method can be used on various fillets and notches problems with curvature constraints.

In Table 3, the techniques used by different researchers for analysis of SCF & its Mitigation has been compared & summarized. This comparison is based taking into consideration the different parameters applied by the researchers like type of loading, material, discontinuity, the mitigation technique etc.

Also, mitigation in SCF obtained by different researchers by applying different mitigation techniques has been compared in Table 3.

## 3. Conclusion

It is observed that sudden change in stress flow lines causes the stress to rise abruptly. Through gradual change in gradient of flow lines mitigation of stress is observed. The rise in the stress concentration factor reaches to its maximum value 3 at the periphery of circular hole. The work compiled above shows that this maximum SCF value can be reduced either by material removal at the vicinity or by shape optimization or by strengthening the hole by inclusion of additional stronger material. Less research work is reported in the area of mitigation of stress concentration in comparison to stress concentration analysis as summarized in Table 2 and Table 3.

Material removal by inclusion of relief holes will be beneficial if the distance between the holes and its diameter is optimum as suggested by the co-author. The material removal method will be more feasible in infinite plates due to the availability of sufficient space around the singularity.

Table 3. Review of stress mitigation in plate with singularities.

| S. No. | Author | Year | Discontinuity | Material | Loading | Analysis Techniques | Mitigation Techniques | Review |
|---|---|---|---|---|---|---|---|---|
| 1 | V. Vicentini [30] | 1967 | Two superposed semi circular notches symmetrically placed at the edges of strip | Isotropic | Pure tension | Analytical Method | Based upon Neuber's equation | Theoretical stress concentration factor are compared with experimental results obtained from strain gauges and photo elasticity in a series of tests performed |
| 2 | K. Rajaiah and N. K. Naik [31] | 1984 | Plate with central circular hole | Isotropic | Tensile uni-axial | Photo elasticity | Hole shape optimization of original and auxiliary hole | Up-to 30% reduction by original hole shape optimization & 23 % reduction by auxiliary hole shape optimization . Feasibility should be decided. |
| 3 | S. A. Meguid [25] | 1986 | Plate with two coaxial holes | Isotropic | Uni-axially Loaded | Finite element method | Defence Hole System | Three systems for defence holes were described. Reduction in maximum SCF ranging from 7.5 to 11 % has been achieved. |
| 4 | H. Engels, W. Hensel, and W. Becker [32] | 2002 | Circular hole | Composite components | Uni-axial , Shear , biaxial | Finite element method | Optimal reinforcement design | The optimal designs obtained in terms of the possible weight reduction and strength improvement |
| 5 | S. Sanyal and P. Yadav [27],[28] | 2005, 2006 | Infinite Thin Plate with Single Circular Hole | Isotropic | Axial Loading | Finite element method | Multiple Relief Holes | Optimum spacing for relief holes and optimum size of relief holes determined. |
| 6 | N. D. Mittal and N. K. Jain [23] | 2007 | Circular hole in square plate | Isotropic, Orthotropic and Composite | Transverse Static Load | Finite element method | Auxiliary hole & cavities | Reduction in SCF depends on size & place of auxiliary hole. |
| 7 | Q. Yang , C.-F. Gao , and W. Chen [32] | 2010 | Plate with a circular hole | Functionally Graded Material | Arbitrary constant loads | Analytical | Proper change ways of the elastic properties | The problem is reduced approx. to the case where the homogeneous plate contains N rings which have different material constants and it can be solved on the basis of Muskhelishvili's theory. |

Optimization of shape of discontinuity for mitigation of stress concentration is not always feasible. Many a times the shape of the singularity cannot be changed because of the design constraint, for example in case of bolt holes, rivet holes etc.

Mitigation of SCF by strengthening the hole is worked out but its feasibility further depends on the method of reinforcement – whether the material will be integral part or will be added separately. Thus, it requires additional manufacturing effort and has to be planned during manufacturing.

Use of functionally graded material for manufacturing the element helps in mitigation of SCF up to some extent.

In the current state of research, it is difficult to affirm which approach seems most suitable to ensure maximum reduction in stress concentration and hence no universal solution or methodology can be proposed for reduction of SCF at the edge of circular hole**.**

The compilation done by Peterson and Roark is commendable but the recent work also should be compiled. The present work is an attempt towards it however the above-mentioned references are by no means exhaustive.

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