ENGINEERING JOURNAL

Article

Analytical and Experimental Investigation of Parameters Affecting Sliding Loss in a Spur Gear Pair

Chakrit Yenti, Surin Phongsupasamit, and Chanat Ratanasumawong*

Department of Mechanical Engineering, Chulalongkorn University, Phayathai Rd., Patumwan, Bangkok 10330, Thailand
* E-mail: chanat.r@chula.ac.th (Corresponding author)

Abstract. The effects of gear module, pressure angle and gear ratio on the sliding losses of a spur gear pair have been investigated analytically and experimentally in this paper. The analytical investigations were done by using the gear meshing model proposed in the authors' former work. The various empirical formulas of friction coefficient are used in the calculation process. The estimated results are compared with the experimental results done by using back-to-back gear test rig. The analytical results agree well with the experimental results. The sliding losses of gears having larger module are higher than the gears having smaller module. The larger pressure angle gears have lower sliding losses, and increasing the gear ratio causes the increase in sliding loss. The estimated results calculated by using the friction coefficient formula proposed by ISO TC60 are the most accurate comparing with the experimental results.

Keywords: Spur gear, sliding loss, module, pressure angle, gear ratio.

ENGINEERING JOURNAL Volume 17 Issue 1
Received 21 May 2012
Accepted 19 July 2012
Published 1 January 2013
Online at http://www.engj.org/
DOI:10.4186/ej.2013.17.1.79

DOI:10.4186/ej.2013.17.1.79

1. Introduction

Increasing efficiency of the transmission gearbox becomes a more important research topic due to the drastically increase in energy demand and the rising of fuel prices. The main source of power loss in a gearbox comes from the large number of transmission gears. Although each gear pair usually has high efficiency up to 99%, multi-stages gear reduction is always used that makes the total power loss increase considerably. Hence the reduction in power loss of each gear pair by only a few percents can reduce the total power loss of the whole transmission system significantly. Therefore the study on the power loss in a spur gear pair is the basic step that is very important for the improvement of gearbox efficiency.

The power loss of a spur gear pair can be categorized into load dependent loss and load independent loss. Load dependent losses are attributed to the sliding and rolling between tooth surfaces during meshing. These losses depend on the tooth surface friction and the sliding velocity. On the other hand, load independent losses can be divided into windage loss or air resistant loss and also oil churning loss for the gear pair using splash lubrication. In the case of small gears operated at low and moderate speeds such as the gears used in passenger vehicles or in farm tractors, the windage loss and the rolling loss are very small comparing with other losses, hence in this case the sliding loss is the dominant power loss affecting the whole efficiency of the gear pair. For this reason, the sliding loss of a spur gear pair is focused in this paper.

To understand and estimate the sliding loss, many researches had been done in the past. Some experimental investigation results were reported by Petry-Johnson *et al.* [1] and Haizuka *et al.* [2]. These researches did many experiments to investigate the effects of various parameters on the power loss of spur gear pairs. Although these are very useful and can be used as the guidelines for gear selection, they did not explain the mechanism of the gear sliding loss hence the amount of sliding loss for various gear parameters and at various driving conditions still cannot be assessed precisely. For this reason there are many researches focused on the analytical method for determining the gear power loss. Y. Michlin and V. Myunster [3] presented the spur gear meshing model to determine the gear sliding loss. This research is very helpful for understanding the mechanism of gear sliding loss, however this model considers the meshing of only one tooth pair and also uses the constant coefficient of friction in calculation that does not match the actual operating condition. The accuracy of the determining method can be improved by using more accurate friction coefficient in calculation process. There are two main methods to determine the friction coefficient: (1) the use of empirical formulas obtained from such twin-disk experiments, and (2) the use of EHL model for determining the friction coefficient. The examples of researches in the first group are the works done by N. E. Anderson and S. H. Loewenthal [4], Y. Terauchi *et al.* [5] and also the previous work of authors [6]. On the other hand, the examples of researches in the second groups are the researches done by H. Xu *et al.* [7], Y. Diab *et al.* [8] and J. Kuria and J. Khiu [9]. Both methods have different merits and demerits. The use of empirical formulas for determining the friction coefficient is easier and more practical to use in the design process, however the uses of these formulas have the restrictions according to their base experimental conditions. On the other hand, the use of EHL model does not have such restrictions and seems to give satisfactorily accurate results, however due to the complication of the method a long computation time must be required. In this paper the analytical method previously presented by authors [6] that is based on the use of several empirical formulas is adopted to investigate the effects of various gear parameters and operating conditions on the sliding loss. The investigations are also done by experiments at various conditions.

2. Estimation of Sliding Loss in a Spur Gear Pair

The sliding loss of a spur gear pair can be estimated analytically by the method previously presented by authors [6], therefore only the main concepts of the method are reviewed here.

2.1. Sliding Loss in a Spur Gear Pair

The sliding loss of a spur gear pair is attributed to the sliding between tooth surfaces during meshing that lead to the friction force along the tooth profile direction. To estimate the sliding loss, firstly the sliding loss of the single tooth meshing is considered, and then the sliding loss during double teeth meshing is determined by combining the sliding loss of single tooth meshing together according to the meshing order.

The sliding loss ratio is the ratio between the sliding loss and the input power as shown by

$$\varphi = \frac{H_3}{H_1} = \frac{H_1 - H_2}{H_1} \tag{1}$$

where H_1 is input power, H_2 is output power and H_3 is power loss.

For single tooth meshing spur gear, the sliding loss ratio in Eq. (1) can be written in terms of gear parameters and meshing position as

$$\varphi = \frac{-n \cdot \tan\alpha \cdot \mu \cdot (1+m)}{1-(n+1)\cdot \tan\alpha \cdot \mu} \tag{2}$$

where n is the position ratio relating to the meshing position,
α is the pressure angle,
m is the speed reduction ratio,
μ is friction coefficient.

A sample of calculated result from Eq. (2) can be shown by a V-line in Fig. 1(a). It is found that the sliding loss ratio is large at the meshing position far away from the pitch point, but it becomes null at the pitch point. The other V-lines in Fig. 1(a) correspond to the other meshing pairs. These V-lines are arranged according to the meshing order. The overlap parts correspond to the moments that double teeth are meshing.

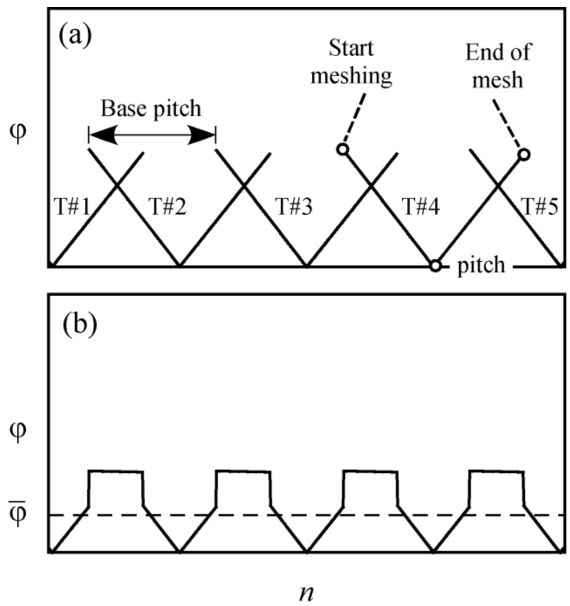

Fig. 1. Sliding loss ratio of a spur gear pair [6].

The power loss ratio during double teeth meshing can be estimated by assuming that both meshing teeth transmit power equally. With this assumption the total sliding loss ratio φ_{total} can be calculated from

$$\varphi_{total} = \frac{\varphi_{Z1} + \varphi_{Z2}}{2} \tag{3}$$

where φ_{Z1} and φ_{Z2} are sliding loss ratios of the first and the second tooth pair respectively. The sliding loss ratio of each meshing pair can also be calculated from Eq. (2). The total sliding loss at any meshing position are plotted and shown in Fig. 1(b). In the figure, the sliding loss varies depending on meshing position. The

DOI:10.4186/ej.2013.17.1.79

sliding loss is large when double teeth are meshing, and becomes least at the pitch point. The dash line in the figure shows the average value of the total sliding loss and can be calculated from

$$\overline{\varphi} = \frac{1}{Pb}\int_{0}^{Pb} \varphi \, dn \tag{4}$$

where Pb is base pitch.

2.2. Friction Coefficient

To calculate the sliding loss in Eq. (2), the value of the friction coefficient must be known. Because many parameters such as rolling and sliding velocity, viscosity of the lubricant, surface roughness and loading condition affect significantly the friction coefficient, the value of friction coefficient must be used to suit with these parameters. There are many researches proposed the empirical formulas for estimation of the friction coefficient [10-14]. These formulas are constructed base on the curve fitting of the results obtained from such twin-disk experiments, and are shown in Table 1. In these formulas, v_k and v_o are kinematic and dynamic viscosities of lubricant, V_s is the relative sliding velocity, V_r is the sum of the rolling velocities, R is the combined radius of curvature, W is the unit normal load, $P_{\max}$ is the maximum contact pressure and S is the surface roughness parameter.

Because these formulas had been constructed experimentally, they have restrictions according to their base experimental conditions. To use these formulas, input parameters in the calculation that are the values of lubricant parameters, surface roughness parameters, and operating parameters are previously checked carefully to assure that these formulas are applicable. Checking results confirm that all formulas in Table 1 are applicable for the experimental conditions here.

Table 1. Empirical formulas used to calculate friction coefficients.

Empirical Formulae	**Published Author**
$\mu = 0.0127\left[\frac{50}{50-S}\right]\mathrm{Log}_{10}\left[\frac{3.17(10)^8 W}{v_o V_S V_r^2}\right]$	Benedict and Kelley
$\mu = \left[0.8\sqrt{v_k}V_S + V_r\varphi + 13.4\right]^{-1}$	Drozdov and Gavrikov
$\varphi = 0.47 - 0.13(10)^{-4}P_{\max} - 0.4(10)^{-3}v_k$	
$\mu = 0.12[WS/(RV_r v_o)]^{0.25}$	ISO TC60
$\mu = 0.325[V_S V_r v_k]^{-0.25}$	Misharin
$\mu = 0.6[(S+22)/35][v_o^{1/8}V_S^{1/3}V_r^{1/6}R^{1/2}]^{-1}$	O'donoghue and Cameron

2.3. Estimation Process

The process for estimation of sliding loss in a spur gear pair is shown in Fig. 2. First gear parameters, lubricant viscosity and operating conditions are input into the computational program. Gear parameters and loading condition are used to calculate the length of contact and the load per length at any arbitrary meshing condition. Calculated load per length from the former procedure is used along with the lubricant viscosity, sliding and rolling velocity, radius of curvature and surface roughness parameter to calculate the friction coefficient at each meshing position. After that the sliding loss ratio of single tooth meshing spur gear can be calculated by using Eq. (2). Total sliding loss ratio can be calculated further by combining the sliding loss ratio of single tooth meshing together according to the meshing order. The sliding loss ratio for the moment that double teeth are meshing can be calculated by using Eq. (3). At last the average sliding loss ratio and sliding loss of a spur gear pair can be calculated.

DOI:10.4186/ej.2013.17.1.79

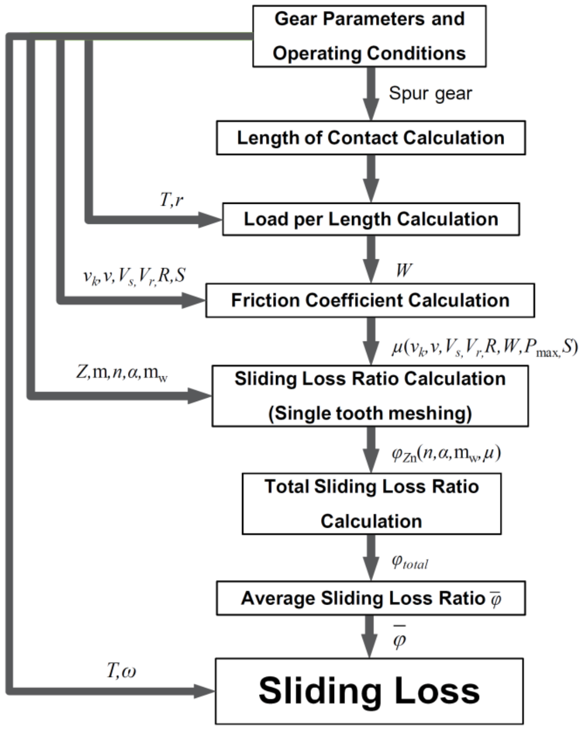

Fig. 2. Diagram of the estimation process.

3. Experiment

3.1. Apparatus

The experiments to investigate the power losses of spur gear pairs in this study were done by using the back-to-back gear test rig as shown in Fig. 3. This type of apparatus does not have any output power. The input power circulates in the system to compensate various kinds of power losses in the system. In other words, the input power is equal to the total power loss in the system. The apparatus composes of 2 identical sets of gears and gear boxes. Two gear boxes are connected together with shafts and couplings. A loading coupling at one side is used to applied load torque into the system by ballasting to twist the shafts to increase the contact forces at the gear tooth surfaces. The load torque applied to twist the shaft was measured by using 4 strain gauges attached at one shaft between gearboxes.

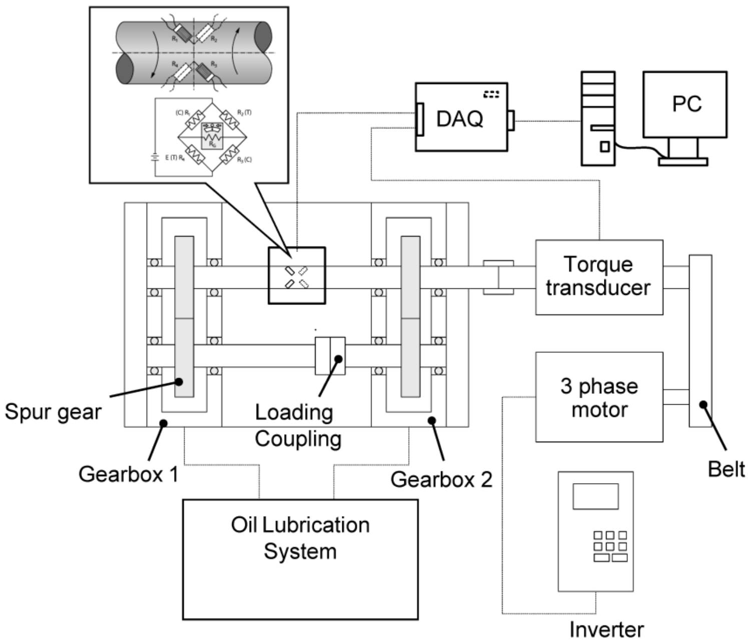

Fig. 3. Experimental apparatus.

A 3 phase induction motor was used to drive the system via the 3:5 pulleys to increase the speed of input shaft. An inverter was used to control the speed of the motor. The jet lubrication system was used in this apparatus. The lubricant oil was 80w90 gear oil. Oil flow rate was controlled at 1 lpm by controlling the rotational speed of oil pump. Because the viscosity of lubricant oil will be changed along with the change of lubrication temperature during operation, to control the lubricant viscosity the temperature of the lubricant was control at 70°C where the change of temperature around this point has only small effect to the change of oil viscosity. In this study the oil temperature was controlled by an immersion heater, and oil temperature was measured by a K-type thermocouple.

The input torque was measured by a torque transducer at the input shaft. The rotational speed of shaft was measured by a tachometer. All signals were amplified and collected by using DAQ system that connects directly to the PC. The product of input torque and rotational speed of shaft is the total power loss of the system.

3.2. Gear Parameters and Operating Conditions

In the experiments, 4 different gear sets that their parameters are shown in Table 2 were prepared to investigate the effects of various gear parameters on the gear sliding loss. The gear design 1 is used as the reference gear set. The gear design 2, 3 and 4 were designed to have different module, pressure angle and gear ratio respectively whereas the other parameters are still the same comparing with gear design 1. All experiments were done at rotational speed 500 to 2500 rpm, load torques at the driven shaft were set between 15 to 250 Nm.

Table 2. Gear parameters.

Parameters	Design 1	Design 2	Design 3	Design 4
Number of teeth	30	45	30	20, 40
Module (mm)	3	2	3	3
Pressure angle (deg.)	20	20	14.5	20
Face width (mm)	20	20	20	20
Pitch diameter (mm)	90	90	90	60, 120
Gear ratio	1:1	1:1	1:1	1:2

3.3. Method and Calculation

Power losses in the back-to-back gear test rig are attributed to 2 main components that are transmission gears and ball bearings as shown in Fig. 4. For the power losses attributed to gears, they can be categorized into the load dependent losses coming from gear meshing that are sliding loss, $P_{sliding}$ and rolling loss, $P_{rolling}$, and the load independent loss coming from air resistance or windage loss, P_w. In the other gear systems, load independent loss may also come from the oil churning resistance or churning loss, however in this experiment the jet lubrication is used hence oil churning resistance does not occur. For the power losses attributed to ball bearings, they can also be categorized to the load dependent loss, $P_{b,\ load\ dep.}$ and load independent loss, $P_{b,\ load\ indep.}$. All load dependent losses attributed to both gears and bearings are called mechanical loss, P_{mech}, on the other hand all load independent losses are called spin loss, P_{spin}.

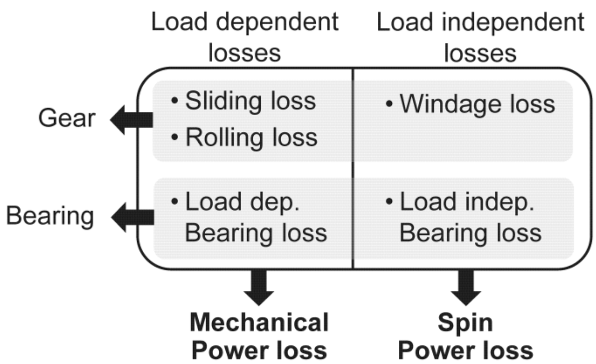

Fig. 4. Category of power losses in the apparatus.

For the experiments that load torque is applied into the system, the total power losses can be written in the terms of various kinds of power losses categorized above as shown in equation

$$P_{total} = P_{mech} + P_{spin} \tag{5}$$

where

$$P_{mech} = (P_{sliding} + P_{rolling}) + P_{b,\,load\,dep.} \tag{6}$$

and

$$P_{spin} = P_w + P_{b,\,load\,indep.} \tag{7}$$

In the case of load torque does not applied into the system, there are just very small contact forces between gear tooth surfaces and also among balls, outer ring or inner ring inside ball bearing, hence the mechanical power losses become very small and can be neglected. In this case the total loss can be considered to be attributed to only spin power loss as shown in equation

$$(P_{total})_{T=0} = P_{spin} \tag{8}$$

Here both experiments with applied load torques and experiments without applied load torques were done. In the first experiment the total power loss shown in Eq. (5) is obtained, whereas the spin power loss in Eq. (8) is obtained from the second experiments. By subtracting the spin power loss in Eq. (8) from the total power loss in Eq. (5) the mechanical power loss is obtained.

In the apparatus there are 2 gearboxes with 8 ball bearings that support gear shafts, and also 4 ball bearings supporting the connecting shafts between two gearboxes, hence the mechanical power loss can be written to match with the actual elements as shown in equation

$$P_{mech} = 2(P'_{sliding} + P'_{rolling}) + [8P'_{b,\,load\,dep.}]_{\text{gearbox}} + [4P'_{b,\,load\,dep.}]_{\text{connectingshaft}} \tag{9}$$

where P' means the power loss of each gear pair or each element.

Because the loads supported by 4 ball bearings at the connecting shafts are mainly attributed to the weights of connecting shaft, they are very small comparing with the loads supported by ball bearing at the gearboxes which also include the loads from the contact forces between gear teeth. With this reason the load dependent losses of ball bearings at the connecting shafts are very small and can be neglected from the calculation. Hence Eq. (9) becomes

$$P_{mech} = 2(P'_{sliding} + P'_{rolling}) + [8P'_{b,\,load\,dep.}]_{\text{gearbox}} \tag{10}$$

The load dependent bearing loss in Eq. (10) can be determined by the method proposed by Harris [15] as shown in equation

$$P'_{b,\,load\,dep.} = \frac{M_b \omega_b}{1000} \tag{11}$$

and

$$M_b = f_1 F_r d_m \tag{12}$$

where ω_b is the rotational speed of ball bearing,

f_1 is the dimensionless coefficient depended on the type of bearing and bearing load,

F_r is the radial force, and

d_m is the average diameter of ball bearing.

Sliding loss that is focused in this paper can be determined by subtracting the load dependent bearing loss calculated from Eqs. (11) and (12) from Mechanical power loss in Eq.(10), moreover from the former researches [3, 9] it is known that the rolling loss is very small comparing with the sliding loss, therefore the result after subtraction can be approximated to be sliding loss. The sliding loss can be calculated from

$$P'_{sliding} = \frac{1}{2} P_{mech} - [4P'_{b,\, load\, dep.}]_{\mathrm{gearbox}} \tag{13}$$

This sliding loss is comparable with the estimated result explained in Section 2.

4. Results and Discussion

4.1. Spin Loss and Mechanical Loss

The results of experiments without applied load are shown in Fig. 5. The measured power losses in these cases are load independent losses or spin losses of the whole system including 2 gear pairs and supported bearings. Although the spin losses are attributed to the rotational resistance of both gears and bearing, the same type of bearing was used in all experiments and the operating conditions in all gear sets were the same, the bearing losses in all cases can be considered to be the same, hence the differences in spin losses in Fig. 5 come from the difference in gear design.

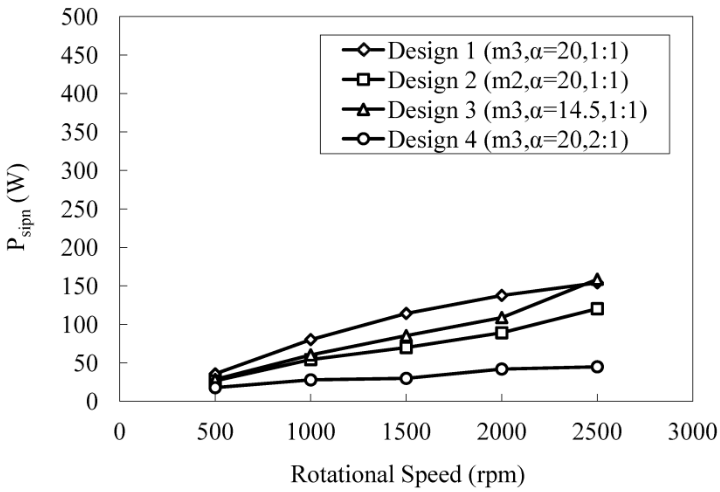

Fig. 5. Measured spin losses.

From the results, the spin losses in all cases increase with rotational speed. The gear designs 1, 2 and 3 that have the same nominal wheel size and the same face width have nearly the same magnitudes of spin losses. However the gear designs 1 and 3 that have larger module have slightly more power losses than the gear design 2. For gear design 4, because the magnitude of spin losses depend on the sizes of gears and the rotational speed, at the same input speed with other cases the gear design 4 has smallest pinion size, although the driven gears are larger than the others, they has slowest output shaft speed, therefore it has the least power losses among the other test gear pairs.

The sample results of the total losses and the mechanical losses of the gear design 1 at various operating conditions are shown in Fig. 6. Total losses in the figure are directly measured from the experiments, whereas the mechanical losses are obtained by subtracting the spin losses shown in Fig. 5 from the total losses. From the figure the power losses increase with load torque and speed. At very small applied torque, the spin loss is the dominant loss, but when the torque is increased the mechanical loss increases and becomes larger than the spin loss. At the large load the mechanical loss is very large comparing with the spin loss and become the dominant loss in the system.

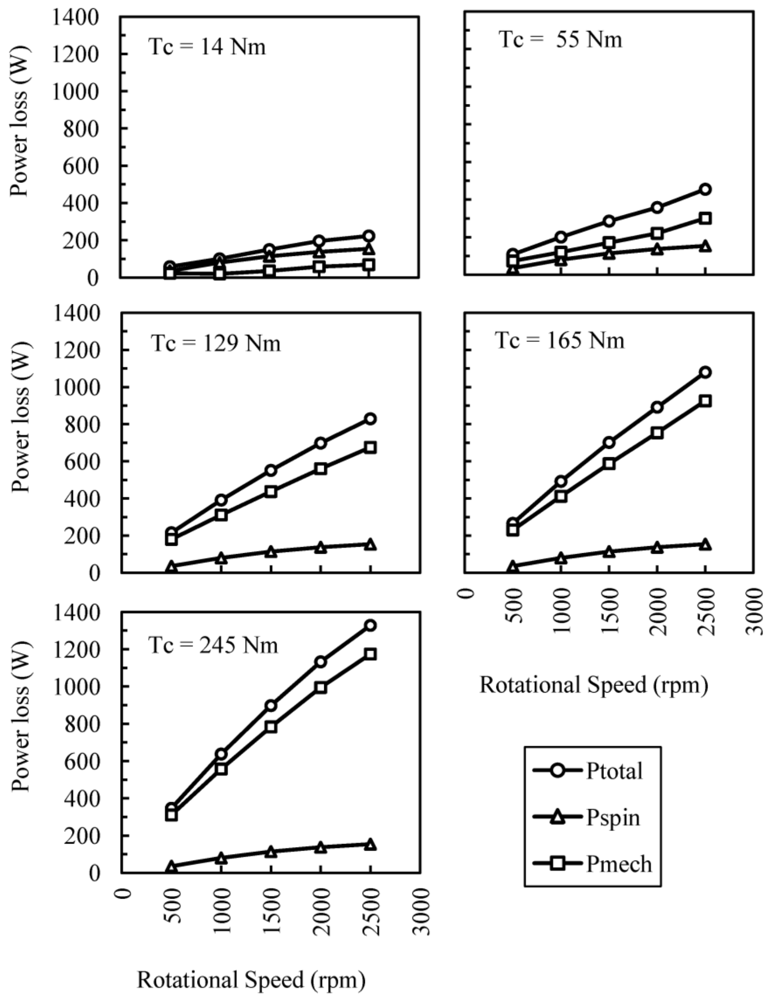

Fig. 6. Measured power losses of gear design 1.

4.2. Sliding Loss

The sliding losses shown in this part are the sliding losses of one gear pair, and are acquired by subtracting the load dependent bearing loss from the mechanical loss as explained in Eq. (13). The sliding losses of various gear designs in various operating conditions are shown in Figs. 7-10. The horizontal axes in these graphs are torques at the input shaft. In detail the applied torques of different gear designs are not exactly the same due to the different gear backlashes, the restrictions in gear assembling procedure, and also the capability of loading couplings that cannot adjust to apply exactly the same load torque as required value. Moreover in the case of gear design 4 that has gear ratio 2:1, the input torque ranges in experiments are also reduced by half comparing with others to keep the same torque range at the driven shaft and also to prevent the damage in the driven shaft.

Figure 7 shows the experimental results of sliding losses for gear design 1-4 operated at various conditions. In the figure, the power losses of all gear pairs are increase almost linearly with speed and load. By considering the magnitude of sliding loss, it is obvious that at rather large load the magnitude of sliding loss is nearly approximate the mechanical loss and is much larger than the magnitude of spin loss. For example at the applied load torque around 200 Nm and rotational speed 2500 rpm, the magnitude of sliding loss of one gear pair in Fig. 7 is about 400 to 500 W therefore the sliding loss of two gear pairs is about 800 to 1000 W, whereas the magnitude of the spin loss of the whole system in Fig. 5 is just less than 200 W. This verifies that during the operation at rather large load as usually found in the normal operation the sliding loss is very significant loss in the gear system.

Figure 8 shows the effect of gear module on the sliding loss that can be known by comparing the sliding losses of gear design 1 that has module 3 mm with those of gear design 2 that has module 2 mm. It

DOI:10.4186/ej.2013.17.1.79

is found from the results that gear design 1 with larger module has higher sliding losses than gear design 2. The differences in sliding losses are small at low speed, but become larger at higher operating speeds.

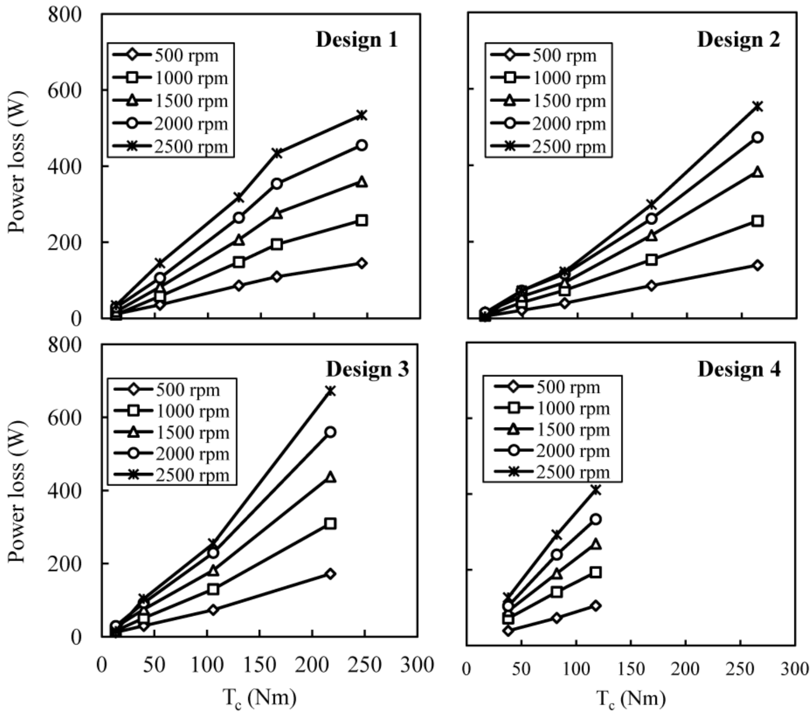

Fig. 7. The effect of operating conditions on the sliding loss.

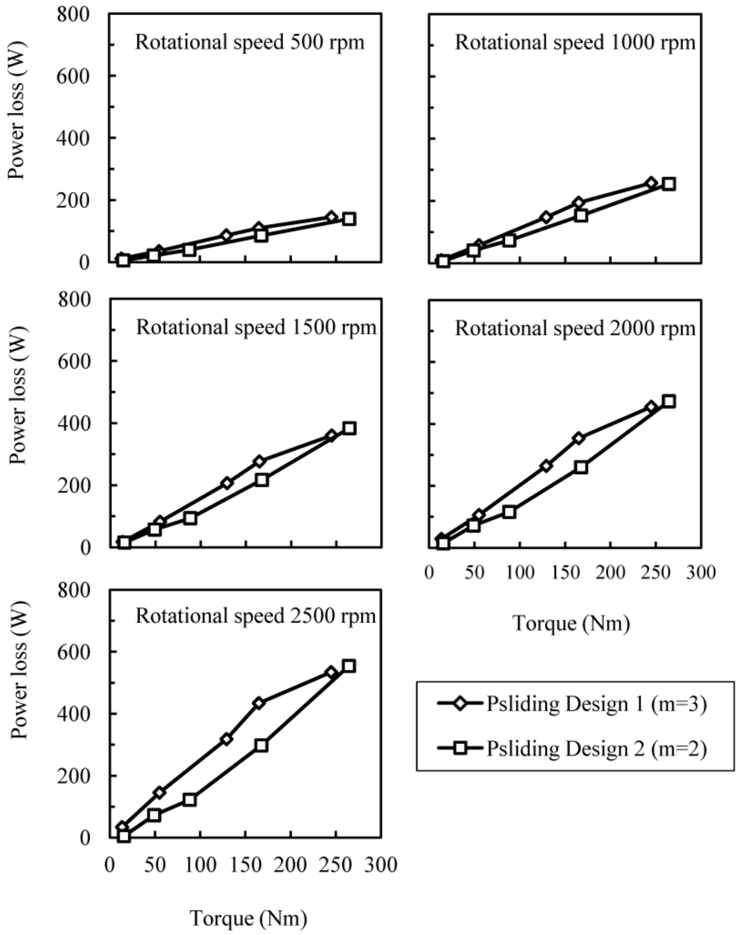

Fig. 8. The effect of module on the sliding loss.

The effect of pressure angle is shown in Fig. 9. These results are obtained by comparing the results in the case of gear design 1 that has pressure angle 20° with the results of gear design 3 that has pressure angle 14.5°. The effect of pressure angle on the sliding loss is not clearly seen when gears are operate at low speed and low torque, but at high torque the sliding losses of gear design 3 become obviously larger than the sliding losses of gear design 2.

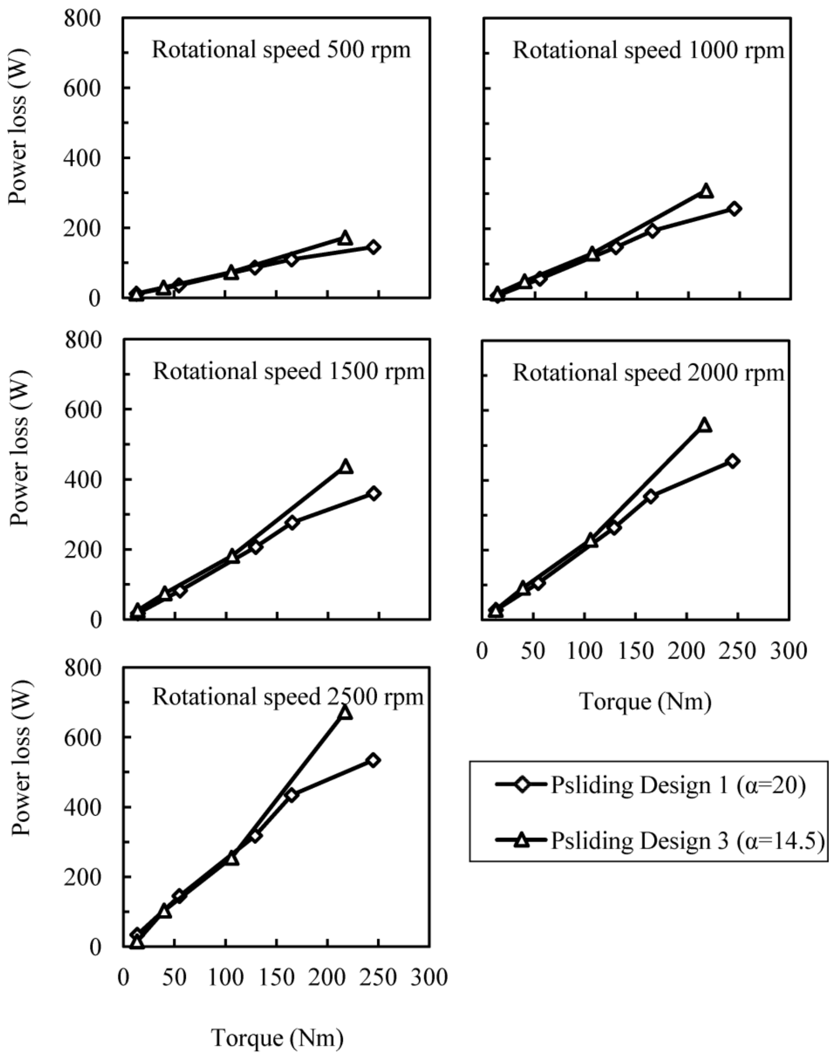

Fig. 9. The effect of pressure angle on the sliding loss.

Figure 10 shows the effect of gear ratio on the sliding loss. At the same input torque range, the sliding losses of gear design 4 that have higher gear ratio are obviously higher than those of gear design 1.

DOI:10.4186/ej.2013.17.1.79

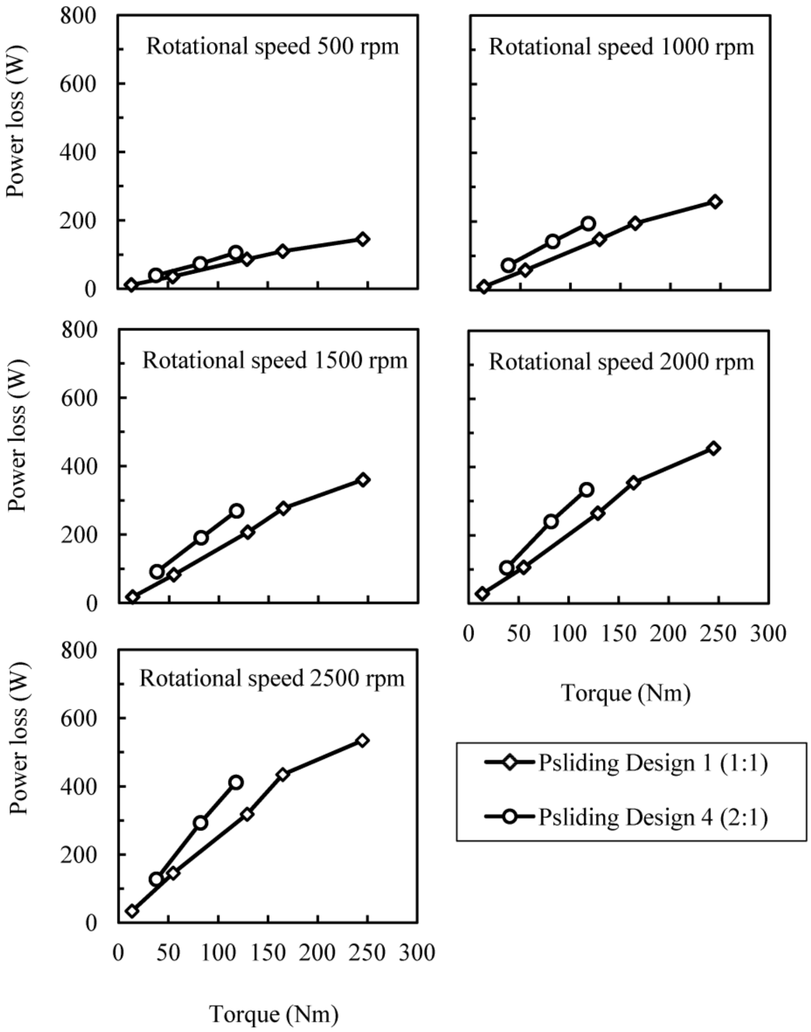

Fig. 10. The effect of gear ratio on the sliding loss.

4.3. Estimated Sliding Loss

The sliding losses estimated using various friction coefficient formulas at various applied torques at input shaft speed 2500 rpm are shown in Fig. 11. All results show the same trend with the experimental results. The gear pair with larger module has higher sliding loss than the gear pair with smaller module. The smaller pressure angle gear pair has higher sliding loss than the larger pressure angle gear pair, and the higher gear ratio has higher sliding loss than the lower one.

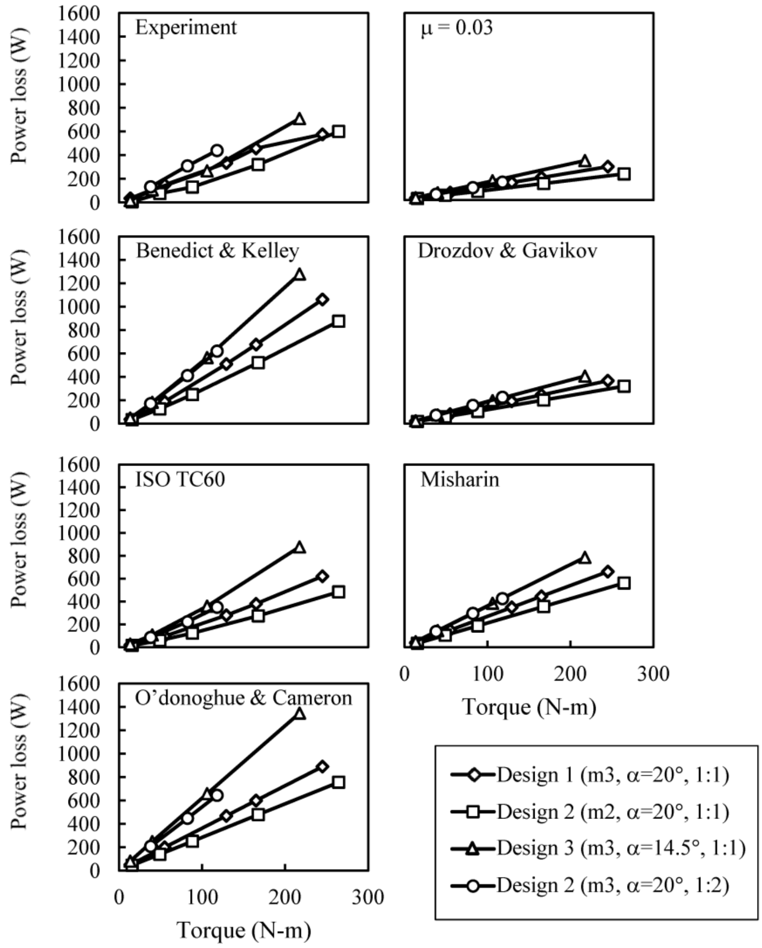

Fig. 11. The estimated sliding loss of various gears at input shaft speed 2500 rpm.

The effect of these parameters on the sliding losses can be explained by considering the estimated sliding loss ratio of single tooth meshing as shown in Fig. 12. Figure 12(a) shows the sliding loss ratio of the gear design 1 and 2. From the figure, the gear pair having larger module have longer meshing length. This means longer distance that friction loss can be occurred. Moreover at the position far away from the pitch point the sliding loss ratio also larger than the position close to pitch point. With these 2 reasons the gear pair with larger module has higher sliding loss than the gear pair having smaller module. The effect of the pressure angle on the sliding loss can also be explained in the same way as the effect of module due to the similar shape of sliding loss ratio as shown in Fig. 12(b). Figure 12(c) shows the effect of gear ratio on the sliding loss ratio. From the figure the meshing lengths of gear ratio 1:1 and 1:2 are almost the same, but at the same meshing position the sliding loss ratio of 1:2 gear pair is larger than 1:1 gear pair. This is because the sliding velocity of 1:2 gear pair is higher than that of 1:1 gear pair at the same meshing position. With this reason the sliding loss ratio of the gear pair having higher gear ratio is higher than the gear pair that has gear ratio 1:1.

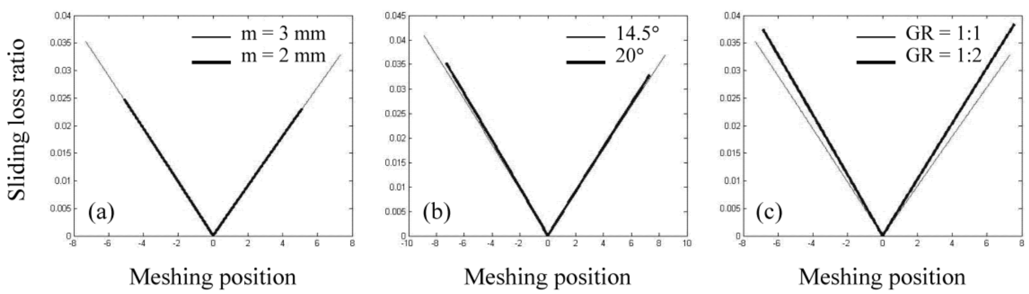

Fig. 12. The sliding loss ratio of single tooth meshing.

DOI:10.4186/ej.2013.17.1.79

Figure 13 shows the comparison between the experimental results and estimated results of gear design 1. It is found that the use of coefficient of friction proposed by ISO TC60 give the most accurate results comparing with the experimental results at almost all operating conditions. On the other hand the results estimated by using the friction coefficient formulas proposed by Benedict and Kelly, O'donoghue and Cameron, and by Misharin are higher than the experimental ones. The estimation using the Drozdov and Gavrikov's formula, and using constant friction coefficient 0.03 also give underestimated results. The estimated results in the other gear pairs also show the same trends with the results shown in Fig. 13 [16].

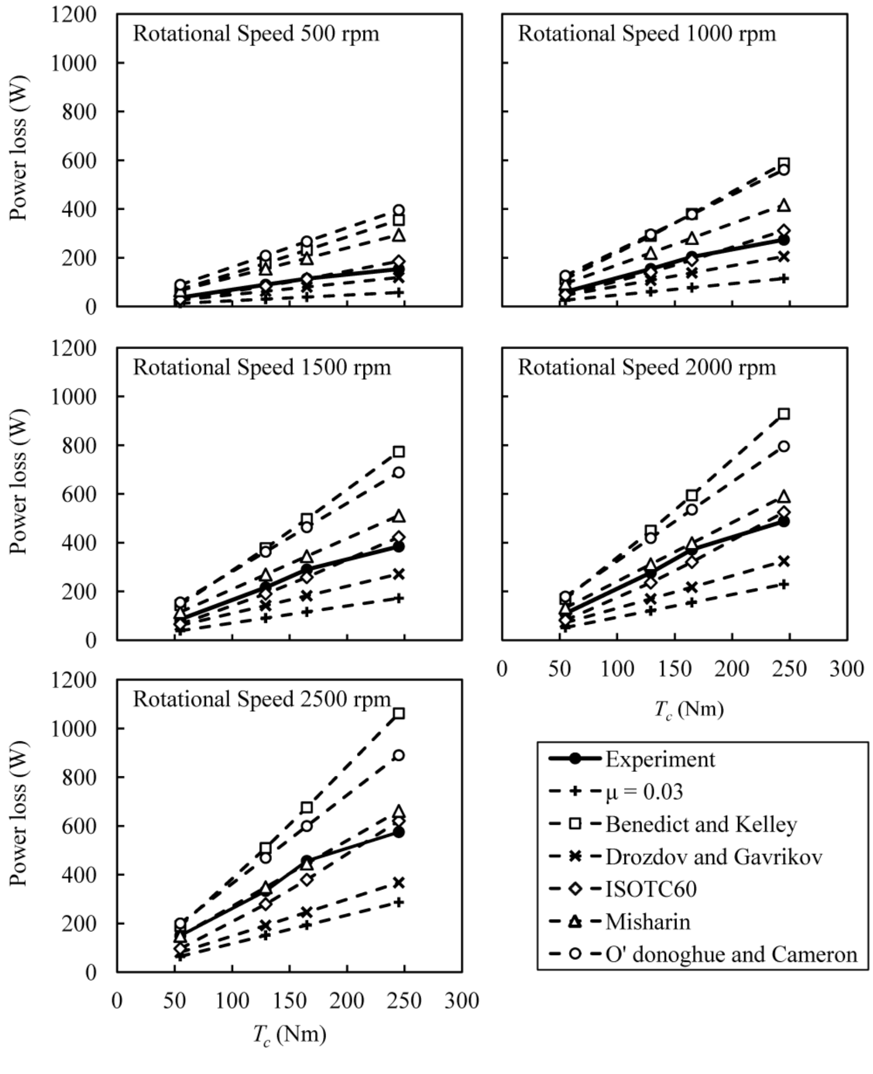

Fig. 13. Comparison between the experimental results and estimated results of gear design 1.

5. Conclusions

The effects of gear parameters and operating conditions on the sliding loss of a spur gear pair have been investigated experimentally and analytically. The sliding losses are increase with increasing of applied load torque and speed. The gear pair with larger module has higher loss than the gear pair having small module. Small pressure angle gear pair has larger sliding loss, and increasing of gear ratio also increase the sliding loss. The estimated results agree well with the experimental results. The use of friction coefficient formula proposed by ISO TC60 in calculation brings the most accurate results.

Acknowledgement

This work was financial supported by The Thailand Research Fund (MRG5480164) and by The Graduate School, Chulalongkorn University.

References

[1] T. T. Petry-Johnson, A. Kahraman, N. E. Anderson, and D. R. Chase, "An experimental and theoretical investigation of power losses of high-speed spur gears," *J. Mech. Des.*, vol. 130, pp. 062601, 2008.

[2] S. Haizuka, T. Kikusaki, and C. Naruse, "Studies on friction loss of spur gears: Effect of viscosity of lubricating oils and tooth forms," *JSME Int. J. C*, vol. 42, no. 4, pp. 1041-1049, 1999.

[3] Y. Michlin, and V. Myunster, "Determination of power loss in gear transmission with rolling and sliding friction incorporated," *Mech. Mach. Theory*, vol. 37, no. 2, pp. 167-174, 2002.

[4] N. E. Anderson, and S. H. Loewenthal, "Effect of geometry and operating conditions on spur gear system power loss," *J. Mech. Des.*, vol. 103, pp. 151-159, 1981.

[5] Y. Terauchi, K. Nagamura, and K. Ikejo, "Study on friction loss of internal gear drives: Influence of pinion surface finishing, gear speed and torque," *JSME Int. J. III*, vol. 34, no. 1, pp. 106-113, 1991.

[6] C. Ratanasumawong, P. Asawapichayachot, S. Phongsupasamit, H. Houjoh, and S. Matsumura, "Estimation of sliding loss in a parallel-axis gear pair," *Special Issue on ICMDT 2011 – J. Adv. Mech. Des. Syst. Manuf.*, vol. 6, no. 1, pp. 88-103, 2012.

[7] H. Xu, A. Kahraman, N. E. Anderson, and D. G. Maddock, "Prediction of mechanical efficiency of parallel axis gear pairs," *J. Mech. Des.*, vol. 129, pp. 58-68, 2007.

[8] Y. Diab, F. Ville, P. Velex, "Prediction of power losses due to tooth friction in gears," *Tribol. Trans.*, vol. 49, no. 2, pp. 260-270, 2006.

[9] J. Kuria, and J. Kihiu, "Prediction of overall efficiency in multistage gear trains," *Int. J. Aerosp. Mech. Eng.*, vol. 5, no. 3, pp. 171-177, 2011.

[10] G. H. Benedict, and B. W. Kelly, "Instantaneous coefficients of gear tooth friction," *ASLE Trans.*, vol. 4, no. 1, pp. 59-70, 1961.

[11] Y. N. Drozdov, and Y. A. Gavrikov, "Friction and scoring under the conditions of simultaneous rolling and sliding of bodies," *Wear*, vol. 11, no. 4, pp. 291-302, 1968.

[12] ISO TC 60, DTR 13989.

[13] Y. A. Misharin, "Influence of the friction condition on the magnitude of the friction coefficient in the case of rollers with sliding," *Proc. Int. Conf. on Gearing, Inst. Mech. Eng.*, London, pp. 159-164, 1958.

[14] J. P. O'Donoghue, and A. Cameron, "Friction and temperature in rolling sliding contacts," *ASLE Trans.*, vol. 9, pp. 186-194, 1966.

[15] T. A. Harris, and M. N. Kotzalas, *Rolling Bearing Analysis – Essential Concepts of Bearing Technology*, Boca Raton, CRC Press, 2007.

[16] C. Yenti, "Measurement and application of a mathematical model for determining power loss of spur gears," *Master of Science Thesis*, Dept. Mech. Eng., Chulalongkorn University, Thailand, 2011 (in Thai).