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## A Unified Optimization Model with Proportional Fairness and Robustness of Fuzzy Multi-Objective Aggregate Production Planning in Supply Chain under Uncertain Environments

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**Abstract.** This study aims to provide Decision Makers (DMs) with a framework for achieving long-term development of Aggregate Production Planning (APP) in a Supply Chain (SC). The contribution of this study is to consider both Proportional Fairness (PF) and robustness in the APP optimization processes, recognizing that overlooking fairness among the multi-objectives in APP may result in inequitable considerations due to different priorities. Neglecting robustness may yield unreliable and non-resilient outcomes in APP, particularly in uncertain situations where uncertain and vague information poses a challenge. To address these concerns, a unified proportional fairness and robustness optimization model is proposed by applying the principles of PF and Robust Chance-Constrained Programming (RCCP) to the conventional weightless objective optimization approach. The effectiveness of this approach is demonstrated through a case study of an APP problem in a SC with the objectives of minimizing total costs, minimizing fluctuations in workforce levels, and maximizing total values of purchasing under uncertain environments. The comparative analysis indicates that the outcome derived from the proposed approach outperforms the results of both traditional weightless and fairness approaches, particularly in enhancing fairness and robustness in the APP.

**Keywords:** Aggregate production planning, supply chain, proportional fairness, robustness.

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## 1. Introduction

Aggregate Production Planning (APP) in a Supply Chain (SC) involves systematic specification of planned levels of production, inventory, subcontracting, etc., in which a time frame typically ranges from 3 to 12 months and, in some instances, could extend up to 18 months [1, 2]. Generally speaking, APP is a comprehensive process designed to align the production objectives of an organization with market demands over an intermediate planning horizon. APP is crucial in supply chain management, serving as a strategic plan to synchronize and optimize production activities. SC consists of five interconnected echelons: suppliers, manufacturers, distribution centers, retailers, and customers, and requires a cohesive plan to ensure the timely and efficient production and distribution of goods. APP bridges all strategic organization goals and operational intricacies of the SC, addressing challenges such as conflicting objectives, uncertainty in data, and the need for coordinated decision-making.

In the intricate supply chain management, DMs generally face two main challenges: conflicting objectives and uncertain data, which significantly shape the effectiveness and performance of the SC. The first challenge involves inherently conflicting objectives when aligning the goals of diverse partners within the SC [3]. For instance, while one partner may prioritize minimizing the total costs, another might emphasize maximizing customer satisfaction. Effectively addressing and reconciling these conflicting objectives is crucial for establishing an efficient SC that meets the diverse needs of its stakeholders. The second challenge involves uncertainty embedded in the data of SC decision-making. These uncertainties can be traced back to two primary sources. Firstly, environmental uncertainty arises from variables such as suppliers' performance and customers' dynamic behavior regarding supply and demand. Fluctuations in these external factors introduce unpredictability that can significantly impact the planning and execution of SC activities. Secondly, system uncertainty emerges from the inherent unreliability of operations and processes within an organization. Inaccuracies or variability in internal processes can introduce further unpredictability into the SC [4]. Effectively managing and mitigating the impact of these uncertainties is imperative to ensure the SC's resilience, adaptability, and success in the face of dynamic market conditions and operational challenges. Consequently, addressing conflicting objectives and uncertain data is a strategic imperative in designing, operating, and continually improving a robust and high-performing SC.

Generally, in developing process of an APP plan, each member in SC usually establishes their own expectation or objective for the APP plan on the basis that it would improve their own performance. Obviously, the expectation from a member is not always aligned with those from other members. This leaves DMs in a difficult situation to prioritize one objective over the others and

hence needs to equally treat them with the same level of respect. In another situation, DMs may, in fact, have no certain priority goal in their judgement at all. As a result, it is necessary for DMs to ensure equal consideration of multiple objectives in the APP in the SC. Hence, methodologies emphasizing the fair treatment of diverse objective functions are consistently employed, with Zimmermann's approach being specifically utilized in this study. Zimmermann's approach, characterized by its weightless fuzzy optimization technique, advocates for the equal consideration of all objective functions. Widely applied to various APP problems in the SC, this approach is designed to yield balanced optimal solutions. However, there are instances where the obtained optimal solution needs to be more balanced despite employing Zimmermann's approach. In particular, after applying Zimmermann's approach, a scenario may arise where the satisfaction level of an objective is disproportionately low compared to other objectives or falls below the set preferences of DMs. Conversely, the satisfaction level of an objective may be excessively too high, resulting in unfair treatment of any interested objectives. Given the challenge posed by the conflicting objectives of APP in SC, the importance of fairness cannot be overstated. When objectives are unfairly prioritized, it can lead to skewed decision-making, favoring certain goals over others. This imbalance may result in suboptimal resource allocation, hindering the APP's efficiency and effectiveness in SC. Unfair treatment of objectives can create an atmosphere of distrust among stakeholders, as those whose interests are neglected may feel marginalized, potentially straining relationships within the supply chain network. In addition, operational inefficiencies may arise when one objective is disproportionately favored, leading to missed opportunities for optimization in other critical areas. Unfairness in decision-making can also amplify the impact of risks and uncertainties, as certain aspects of the APP in SC may be neglected or inadequately addressed, leaving vulnerabilities unattended.

Therefore, Proportional Fairness (PF) principles emerge as a central focal point for effective resolution. The PF criterion, initially introduced by Kelly in 1997 [5], has been extensively examined across diverse disciplines. In the realm of industry, Agnetis et al. [6] delved into its applications, shedding light on its implications and relevance. Furthermore, researchers such as Yu et al. [7] and Cui et al. [8] have conducted comprehensive studies, contributing to a deeper understanding of the proportional fairness criterion's role and impact on communication networks. Additionally, the supply chain domain has witnessed investigations by Mohebbi et al. [9] and Alhusain et al. [10], emphasizing the criterion's versatility and adaptability in addressing challenges within supply chain dynamics. Notably, the conceptual framework of proportional fairness extends beyond its original proposal, as it can be conceptualized as an extension of the Nash solution, particularly within the context of a two-person bargaining game [11]. By incorporating PF into the decision-making process, DMs

can strive to strike a balance that addresses the needs of all stakeholders, fostering collaboration and trust. This ensures that the APP in the SC can mitigate the risk of imbalance and dissatisfaction arising from conflicting goals.

Simultaneously, robustness's importance cannot be overstated because a lack of robustness implies a limited ability to adapt to uncertainties, disruptions, and changes in the business environment, leading to several negative consequences. Without the capability to handle unexpected events or fluctuations, such as sudden changes in demand or supply chain interruptions, the system becomes more prone to breakdowns, delays, and failures. This can decrease customer satisfaction, as orders may be delayed, and product availability may be compromised. Therefore, the principle of robustness plays a crucial role in addressing uncertainties associated with the data distribution of APP in the SC. Robust planning and decision-making involve acknowledging the inherent unpredictability of certain data arising from internal operations and environmental factors. Embracing robustness ensures that the APP in the SC remains resilient and adaptable, capable of accommodating unforeseen variations and disturbances in the dynamic business environment.

By emphasizing fairness and robustness, DMs can lay a foundation for the long-term stability and success of the APP in the SC. Thus, this study encompasses the main contributions as follows:

- The primary contribution lies in developing a unified proportional fairness and robustness optimization model in fuzzy multi-objective APP in SC under uncertain environments. The proposed model represents a pioneering approach that assists DMs in navigating the complexities of the APP in the SC under uncertain conditions.
- The study bridges a significant gap in the existing literature by introducing a novel perspective combining two essential principles, proportional fairness, and robustness, for optimizing multiple APP objectives in the SC. The integration of proportional fairness and robustness yields substantial benefits for the efficiency and sustainability of supply chain management. Fairness ensures equitable treatment among diverse stakeholders, preventing biases and fostering positive relationships within the SC. At the same time, robustness assists in navigating unforeseen challenges and disruptions, ensuring operational continuity. Ultimately, the incorporation of fairness and robustness fortifies operational resilience and strengthens corporate reputation by signaling a commitment to responsible and reliable business practices.
- The proposed approach clearly outperforms the traditional weightless fuzzy optimization approach when there is a significant disparity of the satisfaction levels among objectives. For instance, the satisfaction level of one objective is

disproportionately low compared to other objectives or even falls below the set preferences of DMs. Conversely, the satisfaction level of another objective may be excessively high, resulting in unfair treatment of any interested objectives.

The subsequent sections of this paper are structured as follows: Section 2, titled "**Literature Review**," provides an extensive review encompassing broad definitions and perspectives from relevant sources on the topics under consideration. Section 3, entitled "**Research Problem Formulation**," introduces a numerical case illustrating the application of the multi-objective APP in SC under uncertain environments. This case study serves as a practical evaluation, assessing the efficacy and applicability of the proposed approach. Section 4, "**Mathematical Formulation**," describes fuzzy multi-objective APP in a SC problem, underlying assumptions, and the associated model formulation. Section 5, denoted as "**Methodology Framework**," a comprehensive exposition provided on the unified proportional fairness and robustness optimization approach, elucidating its intricacies and detailing the procedures employed for its resolution. Section 6, designated as "**Results and Discussions**," disseminates the computational results, accompanied by a thorough analysis, discussions, and managerial implications derived from the findings of this study. Finally, Section 7, titled "**Conclusions**," encapsulates the concluding remarks, delineates any identified limitations, and outlines potential avenues for future research. The structured organization of the remaining sections aims to facilitate a coherent and systematic presentation of the study.

## 2. Literature Review

The first section of the literature review centers on relevant studies, offering an in-depth analysis of APP in SC-related topics and the theoretical frameworks of optimization with proportional fairness and robustness. This section aims to comprehensively understand APP's foundational principles and prior research efforts in SC, fairness, and robustness. It establishes the conceptual groundwork for contextualizing the unified fuzzy optimization approaches proposed in the study.

Multiple Conflicting Objectives of APP problems in SC under uncertain environments have seen significant attention due to the complexities associated with simultaneous optimization and the inherent unpredictability in supply chain dynamics. Traditional approaches to APP optimization have been extended to consider multiple, often conflicting, objectives to capture organizations' diverse goals better. The initial work by Nahmias [12] laid the groundwork by acknowledging the challenge of conflicting objectives in APP, and subsequent research has expanded upon this foundation to incorporate uncertainty considerations. Parames and Kanokbhorn [13] studied the cockpit crew rostering problem in a low-cost airline that considers four objectives; minimization of nautical mile cost, minimization of

Integrating multiple conflicting objectives with uncertainty poses unique challenges in APP within SCs. Gen et al. [14] introduced an efficient approach to convert a fuzzy multi-objective linear programming problem model into a crisp multi-objective linear programming model and proposed an interactive solution methodology that suggests the best compromise aggregate production plans for the multi-period fuzzy multiple objective APP problems. Wang and Liang [15] then developed a fuzzy multi-objective linear programming (FMOLP) model for solving the multi-product APP decision problem in an imprecise environment. Their proposed model attempts to minimize total production costs, carrying and back ordering costs, and rates of changes in labor levels considering inventory level, labor levels, capacity, warehouse space, and the time value of money. Liang and Cheng [16] proposed a two-phase fuzzy goal programming method to solve multiple objectives APP problems with multiple products and periods. The designed fuzzy multi-objective linear programming model aims to minimize total costs simultaneously, total carrying and back ordering volume, and total rates of changes in labor levels associated with inventory carrying levels, machine capacity, workforce levels, warehouse space, and available budget. Ramezani et al. [17] applied evolutionary algorithms to solve multiple objective APPs under uncertainty, aiming to balance conflicting objectives while considering uncertainties in demand, supply, and other operational parameters. Their study recognized the necessity of adapting production plans to supply chain environments' dynamic and uncertain nature.

Generally, circumstances in certain, DMs attempt to balance all objectives of all echelons in the SC. The significance of assigning equal importance to objective functions of APP in the SC lies in its capacity to promote fairness, balance, and overall optimization without prioritizing one objective over the others. In the intricate landscape of supply chain management, where multiple objectives, such as cost minimization, fluctuation in workforce level minimization, and total values of purchasing maximization, treating these goals equally ensures a well-rounded and unbiased decision-making process. Equally weighting objective functions guards against the potential bias that might arise from favoring one specific metric at the expense of others, thus promoting fairness and transparency. This equilibrium is vital for achieving a comprehensive and sustainable strategy, as it acknowledges the interconnected nature of various business objectives. By equally weighing these functions, decision-makers can foster a holistic perspective that enhances overall performance and aligns with broader organizational values and stakeholder expectations. This approach contributes to improved decision quality, stakeholder satisfaction, and the long-term viability of the business.

Traditional APP models often fail to capture the intricate trade-offs necessary when considering multiple conflicting objectives. Therefore, scholars have increasingly recognized the need to develop fairness

models that account for balancing all needs of stakeholders' objectives. The fairness of the APP in the SC refers to the ethical and equitable treatment of various elements within the supply chain management system. It considers diverse objectives, stakeholders, and factors involved in decision-making. A fair APP ensures that no single objective is disproportionately prioritized, recognizing the interconnectedness of multiple goals, such as minimizing costs, minimizing fluctuation in workforce level, and maximizing total values of purchasing. In essence, a fair APP in supply chain management strives to create an environment where decisions are made judiciously, considering the supply chain's multifaceted nature and fostering collaboration, trust, and long-term sustainability. Many research works about the fairness model, such as Naldi et al. [18], investigated how profit optimization can be sought while simultaneously achieving the desired level of fairness. Adopting a maximin approach to fairness and using an Integer Linear Programming (ILP) solver showed that a linear trade-off is possible, since fairness and profit exhibit a nearly perfect linear anticorrelation. Fairness could be improved by even a small profit reduction, especially in large companies (i.e., managing many projects). Hamid et al. [19] studied the trade-off between the costs and the fairness of a collaborative production planning problem in Make-To-Order (MTO) manufacturing. They proposed a mixed-integer linear production planning problem with multiple periods and items specifications in a MTO manufacturing system. Liu et al. [20] investigated how retailers' fairness concerns affect cooperative relationships in a Three-Party Sustainable Supply Chain (TSSC) and how to coordinate such a SC when the degree of fairness concern is treated as an interval. Their study sought equilibrium solutions and profits under five cooperative and non-cooperative models. It revealed that fairness concerns affect members' decisions for sustainable supply chain management. Chen et al. [21] adopted economic efficiency as its main consideration, used specific Emission Reduction Measures (ERMs) of industrial enterprises as minimum allocation units, and constructed an Enterprise-level Pollutant Emission Reduction Allocation (EPERA) model with minimization of the Total Abatement Cost (TAC) as the objective function, and fairness and feasibility as constraints for emission reduction allocation.

In addition, the traditional APP models have even been exacerbated by the inherent uncertainties in the APP of the SC operations. Thus, the robust models are extended to account for the variation in results due to uncertainty. The robustness of the APP in the SC refers to the system's ability to maintain stability and functionality under various conditions and uncertainties. In the context of supply chain management, a robust APP is designed to withstand disruptions, adapt to changes, and continue operating efficiently. This resilience is crucial in the face of dynamic challenges such as market fluctuations, supply chain disruptions, and unexpected events. A robust APP can handle uncertainties by incorporating flexibility and adaptability into decision-making processes. It considers

diverse scenarios, anticipates potential risks, and ensures that the SC can respond effectively to unforeseen circumstances. Robustness extends to optimizing resources, allowing for balanced and effective allocation even in the presence of uncertainties. Ultimately, a robust APP in supply chain management contributes to long-term stability, providing the capacity to navigate complexities and uncertainties and enhancing the overall resilience of the SC. Some researchers have been concerned about the model's robustness. Leung et al. [22] proposed a robust optimization model for solving a multi-site production planning problem for a multinational lingerie company in Hong Kong with uncertain data in which the total costs of production, labor, inventory, and workforce changing costs are minimized. Rahmania et al. [23] developed a new robust fuzzy approach to formulating the APP, where some parameters, such as production cost and customer demand, are fuzzy. Entropy was used to reduce the sensitivity of noisy data and obtain a more robust aggregate production plan. Zhang et al. [24] presented an inexact, robust two-stage MILP approach for crop area planning under uncertainty. Their approach was developed by incorporating the techniques of interval parameter programming, robust optimization method, and MILP within a two-stage stochastic programming optimization framework. Singh and Biswal [25] proposed multiple objectives, multiple product production planning model of a captive repair shop for overhauling and repairing products or machines, indicating the theoretical performance's relevance. Their model can handle robust optimization problems under uncertainty where the solution is satisfied for any possible realization of the data in the uncertainty set. Daryanian et al. [26] developed a new fuzzy, robust stochastic model to deal with the crisis logistics of medicinal products by considering the dimensions of sustainability and resilience using multi-objective programming. Their model has sought to find the optimal place to cover the demand when establishing field hospitals, the optimal route for transporting pharmaceutical products from the drug collection stands to the pharmacy, and the optimal route for transportation from the accident site to the field hospitals.

However, there needs to be more existing research on integrating proportional fairness and robustness within a fuzzy linear programming approach for solving APP problems in SCs under uncertain environments, as summarized in Table 1. The summary highlights a noteworthy area for potential investigation and exploration. According to the reviewed literature on the APP in the SC and fuzzy optimization approaches, it was found that there is no specific research that has proposed a combinatorial fuzzy optimization technique for designing a multiple-objective APP in the SC problems to be both fair and robust under uncertain environments. Therefore, this study developed a unified proportional fairness and robustness fuzzy optimization approach for handling the enumerated obstacles of the APP in the SC and yielding the proportional fairness and robustness optimally. As a result, the proposed approach can support

decision-makers in obtaining an APP plan that is more effective, informative, and compatible with a real-life environment.

### 3. Research Problem Formulation

A supply chain optimization within the context of the APP problem involves four qualified suppliers, who supply raw materials, a single manufacturer responsible for production, and customers, as illustrated in Fig. 1. The plan is set for a 6-month planning period. This optimization approach encompasses three primary objective functions: firstly, the minimization of total costs, taking into account all cost elements such as raw material, production, and related costs; secondly, the minimization of fluctuations in workforce levels, aiming to achieve a stable and efficient workforce allocation; and thirdly, the maximization of total values in purchasing, focusing on optimizing the highest amount of purchased raw materials. This optimization is conducted in an uncertain environment where parameters such as customer demand, product failure rates, service levels, and associated costs are subject to variation represented by Triangular Fuzzy Numbers (TFN). The multifaceted nature of this optimization challenge necessitates a comprehensive approach to address the diverse objectives and uncertainties inherent in the supply chain dynamics.

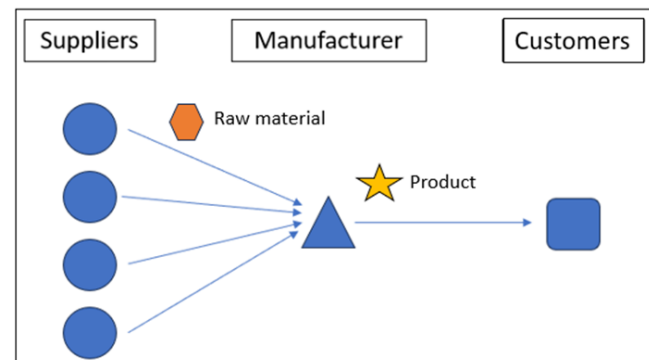


Fig. 1. The structure of SC.

#### 3.1. Problem Assumptions

A group of certified suppliers has been identified, and their performance has been assessed and rated based on criteria such as price, quality, and service level, as presented in Table 2.

Table 2. Performance of Suppliers.

Supplier	Criteria			Score of Supplier ( $TS_s$ )
	Price	Quality	Service Level	
1	Expensive	Excellent	Excellent	0.44
2	Medium	Low	Good	0.20
3	Cheap	Low	Low	0.14
4	Medium	Good	Low	0.22

Table 1. Summary of the literature on APP problems in SC.

Articles	Types of Problems	Types of Parameters	Number of objectives	Uncertain Situation	Types of Model	Properties of Model			
						Fairness Property	Robustness Property	Defuzzification Property	Compromising Property
Al-e-Hashem et al. [27]	APP	D+TFN	M	/	MOFLP	-	/	/	/
Pishvae et.al. [28]	SCND	D	M	-	MILP	-	/	-	/
Rezakhani [29]	CP	D+TFN	S	/	MILP	-	-	/	-
Niknamfar et al. [30]	APP with DP	D+TFN	S	/	MILP	-	/	/	-
Modarres and Izadpanahi [31]	APP	D+TFN	M	/	MOFLP	-	/	/	/
Entezaminia et al. [32]	GSC	D+TFN	S	/	MILP	-	/	/	-
Chutima and Yothaboriban [33]	PAL	D+TFN	M	/	MOFLP	-	-	/	/
Liu and Papageorgiou [34]	SCP	D	S	-	MILP	/	-	-	-
Hormozi et al. [35]	APP	D	M	-	MILP	/	-	-	/
Chutima and Kirdphoksap [36]	CSP	D	M	-	MILP	-	-	-	/
Chutima and Arayikanon [37]	ACCR	D	M	-	MILP	-	-	-	/
Tuan and Chiadamrong [38]	APP	D+TFN	M	/	MOFLP	-	/	/	/
Esteso et al. [39]	SCP	D	M	-	MILP	/	-	-	/
Tirkolae et al. [40]	SAPP	D+TFN	M	/	MOFLP	-	/	/	/
Morais [41]	EVCM	D	S	-	MILP	/	-	-	-
This study	APP in SC	D+TFN	M	/	MOFLP	/	/	/	/

**Abbreviations:** M = Multiple Objectives, S = Single Objective, D = Deterministic Number, TFN = Triangular Fuzzy Number, MOFLP = Multiple Objectives Fuzzy Linear Programming, MILP = Mixed Integer Linear Programming, SCND = Supply Chain Network Design, CP = Construction Projects, DP = Distribution Planning, GSC = Green Supply Chain, PAL = Parallel Assembly Line, CSP = Car Sequencing Problems, ACCR = Airline Cockpit Crew Rostering, SCP = Sustainable Crop Planning, SAPP = Sustainable Aggregate Production Planning, EVCM = Electric Vehicles Charging Management

With multiple criteria to be judged, multi-criteria ranking and scoring methods such as the Analytic Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) can be used to determine the best alternative from a set of options. In this study, TOPSIS was used to determine the preference score of each supplier to calculate the total values of purchasing, as seen in Table 2. For the sake of brevity, the detailed calculations of each supplier's score are not presented here. Other model assumptions are as follows:

- The uncertainty in the failure rate of raw materials and the manufacturer's service level arises from potential defects and on-time delivery variability, respectively.

- Customer demands for products fluctuate over a 6-month planning period, and all associated costs within the SC are subject to be uncertain.
- The fulfillment of product demand may result in either satisfactory completion or shortages.
- However, any shortage incurs a penalty in the form of shortage costs.
- Delivery lead time is considered negligible.

Tables 3-4 display input parameters for the APP in the SC model, encompassing both precise and fuzzy values. For this illustration, three significant points of the Triangular Fuzzy Numbers (TFN) are derived by introducing variations of  $\pm 20\%$  from the most likely value.

Table 3. Precise Parameters.

Parameters	Values						Units
$IWO$	10						persons
$P$	65						%
$AWV$	15						%
$CapR$	10,000						units
$CapP$	3,000						units
$ProdT$	0.4						minutes
$RPP$	5						units
	Period (t)						
	1	2	3	4	5	6	
$ART_t$	144	160	168	176	120	192	hours
$AOT_t$	50	50	50	60	40	60	hours
$MaxM_t$	250	250	250	250	250	250	m/c-hours
$MU_t$	0.5	0.5	0.5	0.5	0.5	0.5	m/c-hours/unit
$WSR_t$	7	7	7	7	7	7	m <sup>2</sup> /unit
$WSP_t$	3.5	3.5	3.5	3.5	3.5	3.5	m <sup>2</sup> /unit
$MaxWS_t$	5,000	5,000	5,000	5,000	5,000	5,000	m <sup>2</sup>
	Period (t)						
	1	2	3	4	5	6	
	Supplier (s)						
	1	3,500	3,500	3,500	3,500	3,500	3,500
	2	3,000	3,000	3,000	3,000	3,000	3,000
$MaxR_{s,t}$	3	3,500	3,500	3,500	3,500	3,500	3,500
	4	3,000	3,000	3,000	3,000	3,000	3,000
							units

Table 4. Fuzzy Parameters (most likely values).

Parameters	Values						Units
$\overline{ACE}$	1.2						%
	Period (t)						
	1	2	3	4	5	6	
$\overline{CRT}_t$	0.6	0.6	0.6	0.6	0.6	0.6	\$
$\overline{COT}_t$	1.2	1.2	1.2	1.2	1.2	1.2	\$
$\overline{WS}_t$	150	150	150	150	150	150	\$
$\overline{HC}_t$	50	50	50	50	50	50	\$

Parameters		Values						Units
		Period (t)						
		1	2	3	4	5	6	
$\widetilde{FC}_t$		70	70	70	70	70	70	\$
$\widetilde{ACSL}_t$		0.7	0.7	0.7	0.7	0.7	0.7	%
$\widetilde{ICR}_t$		1.8	1.8	1.8	1.8	1.8	1.8	\$
$\widetilde{ICP}_t$		4.59	4.59	4.59	4.59	4.59	4.59	\$
$\widetilde{TCP}_t$		8.4	8.4	8.4	8.4	8.4	8.4	\$
$\widetilde{PeC}_t$		2.8	2.8	2.8	2.8	2.8	2.8	\$
$\widetilde{De}_t$		2,510	4,320	1,630	3,440	1,250	2,460	units
		Period (t)						
		1	2	3	4	5	6	
$\widetilde{TCR}_{s,t}$	Supplier (s)							
	1	1	1	1	1	1	1	
	2	0.6	0.6	0.6	0.6	0.6	0.6	\$
	3	0.3	0.3	0.3	0.3	0.3	0.3	
	4	0.6	0.6	0.6	0.6	0.6	0.6	
		Period (t)						
		1	2	3	4	5	6	
$\widetilde{PuC}_{s,t}$	Supplier (s)							
	1	2	2	2	2	2	2	
	2	1	1	1	1	1	1	\$
	3	0.5	0.5	0.5	0.5	0.5	0.5	
	4	1	1	1	1	1	1	
		Supplier (s)						
		1	2	3	4			
$\widetilde{AVSL}_s$		0.8	0.75	0.7	0.7			%
$\widetilde{AVFR}_s$		0.009	0.015	0.015	0.015			%

4. Mathematical Formulation

4.1. Mathematical Notations

All indices, parameters, and decision variables are explicitly defined, with fuzzy parameters denoted by a tilde symbol ( $\widetilde{\cdot}$ ).

Indexes

- $s$  index of suppliers,  $s = 1, 2, \dots, S$
- $t$  index of period,  $t = 1, 2, \dots, T$

Parameters

- $IW_0$  Initial number of workers (persons)
- $P$  Worker productivity (%) ( $0 \leq P \leq 1$ )
- $AWV$  Acceptable worker variation (%)
- $TS_s$  Total score of suppliers  $s$  (%)
- $ProdT$  Production time for producing a product at the manufacturing plant (min)
- $ART_t$  Available regular time in period  $t$  (hours)
- $AOT_t$  Available over time in period  $t$  (hours)

- $RPP$  Number of raw materials needed to produce a product (units)
- $MaxM_t$  Maximum machine capacity in period  $t$  (m/c-hrs)
- $MU_t$  Machine hourly usage for a product in period  $t$  (m/c-hrs/unit)
- $WSR_t$  Warehouse space for raw materials at the manufacturing plant in period  $t$  (m<sup>2</sup>/unit)
- $WSP_t$  Warehouse space for products at the manufacturing plant in period  $t$  (m<sup>2</sup>/unit)
- $MaxWS_t$  Maximum warehouse space at the manufacturing plant in period  $t$  (m<sup>2</sup>)
- $MaxR_{s,t}$  Maximum capacity of raw materials provided by supplier  $s$  in period  $t$  (units)
- $\widetilde{CRT}_t$  Fuzzy cost of regular-time production in period  $t$  (\$/min)
- $\widetilde{COT}_t$  Fuzzy cost of overtime production in period  $t$  (\$/min)
- $\widetilde{WS}_t$  Fuzzy workers' salary in period  $t$  (\$/person)



$\widetilde{HC}_t$	Fuzzy hiring cost in period $t$ (\$/person)
$\widetilde{FC}_t$	Fuzzy firing cost in period $t$ (\$/person)
$\widetilde{ACSL}_t$	Fuzzy acceptable service level of the manufacturing plant in period $t$ (%)
$\widetilde{ICR}_t$	Fuzzy inventory cost of raw materials in period $t$ (\$/unit)
$\widetilde{TCR}_{s,t}$	Fuzzy transportation cost of raw materials from supplier $s$ in period $t$ (\$/unit)
$\widetilde{AVSL}_{s,t}$	Fuzzy average service level of supplier $s$ in period $t$ (%)
$\widetilde{TCP}_t$	Fuzzy transportation cost of products from the manufacturing plant to customers in period $t$ (\$/unit)
$\widetilde{ICP}_t$	Fuzzy inventory cost of products in period $t$ (\$/unit)
$\widetilde{PeC}_t$	Fuzzy penalty cost of product shortage for customers in period $t$ (\$/unit)
$\widetilde{De}_t$	Fuzzy customer demands of products in period $t$ (units)
$\widetilde{ACFR}$	Fuzzy acceptable failure rate of raw materials at the manufacturing plant (%)
$\widetilde{AVFR}_s$	Fuzzy average failure rate of raw materials supplied from supplier $s$ (%)
$\widetilde{PuC}_{s,t}$	Fuzzy purchasing cost of raw materials provided from supplier $s$ in period $t$ (\$/unit)
<b>Decision Variables</b>	
$NW_t$	Number of workers in period $t$ (persons)
$NHW_t$	Number of hired workers in period $t$ (persons)
$NFW_t$	Number of fired workers in period $t$ (persons)
$IR_t$	Inventory of raw materials at the end of period $t$ (units)
$RTQ_t$	Quantity of products produced in regular time in period $t$ (units)
$OTQ_t$	Quantity of products produced in overtime in period $t$ (units)
$PQ_t$	Quantity of products for customers in period $t$ (units)
$IP_t$	Inventory of products at the end of period $t$ (units)
$SPQ_t$	Shortage of products for customers in period $t$ (units)
$RQ_{s,t}$	Quantity of raw materials provided by supplier $s$ in period $t$ (units)

## 4.2. Mathematical Model

In this study, the formulation of a fuzzy multi-objective APP in SC is presented as follows:

### 4.2.1. Objective functions

1. **Minimization of total costs** is often considered a primary objective when describing an effective APP in SC strategy. Typically, the total costs ( $\widetilde{TC}$ ) may exhibit uncertainty, representing the sum of purchasing cost

( $\widetilde{TPuC}$ ), production cost ( $\widetilde{TPrC}$ ), workers' costs ( $\widetilde{TWC}$ ), inventory cost ( $\widetilde{TIC}$ ), transportation cost ( $\widetilde{TTC}$ ), and shortage cost ( $\widetilde{TSQ}$ ) over a specified time period, as outlined in Eqs. (1) – (7).

$$\text{Minimize } \widetilde{TSC} = \widetilde{TPuC} + \widetilde{TPrC} + \widetilde{TWC} + \widetilde{TIC} + \widetilde{TTC} + \widetilde{TSQ} \quad (1)$$

such that:

$$\widetilde{TPuC} = \left( \sum_{s=1}^S \sum_{t=1}^T \widetilde{PuC}_{s,t} \times RQ_{s,t} \right) \quad (2)$$

$$\widetilde{TPrC} = \left( \sum_{t=1}^T \widetilde{CRT}_t \times PT \times RTQ_t \right) + \left( \sum_{t=1}^T \widetilde{COT}_t \times PT \times OTQ_t \right) \quad (3)$$

$$\widetilde{TWC} = \left( \sum_{t=1}^T \widetilde{WS}_t \times NW_t \right) + \left( \sum_{t=1}^T \widetilde{HC}_t \times NHW_t \right) + \left( \sum_{t=1}^T \widetilde{FC}_t \times NFW_t \right) \quad (4)$$

$$\widetilde{TIC} = \left( \sum_{t=1}^T \widetilde{ICR}_t \times IR_t \right) + \left( \sum_{t=1}^T \widetilde{ICP}_t \times IP_t \right) \quad (5)$$

$$\widetilde{TTC} = \left( \sum_{s=1}^S \sum_{t=1}^T \widetilde{TCR}_{s,t} \times RQ_{s,t} \right) + \left( \sum_{t=1}^T \widetilde{TCP}_t \times PQ_t \right) \quad (6)$$

$$\widetilde{TSQ} = \left( \sum_{t=1}^T \widetilde{PeC}_t \times SPQ_t \right) \quad (7)$$

2. **Minimization fluctuations in workforce levels (FW)** are crucial, as maintaining a consistent workforce level is challenging. Excessive fluctuations in worker levels may result in the company losing out on the benefits of skilled workers and incurring substantial compensatory costs. Therefore, stability in worker levels is imperative.

$$\text{Minimize } \text{TCNW} = \sum_{t=1}^T \text{NHW}_t - \text{NFW}_t \quad (8)$$

3. **Maximization of total values of purchasing (TVP)** is crucial, as it ensures that the company acquires the highest quantity of raw materials from suppliers with the top qualifications in terms of price, quality, and timely delivery.

$$\text{Maximize } \text{TVP} = \left( \sum_{s=1}^S \text{TS}_s \times RQ_{s,t} \right) \quad (9)$$

Please be noted that each supplier's performance can be assessed using the TOPSIS as presented in Table 2.

### 4.2.2. Constraints

1. **Raw material quality assessment:** This criterion serves as a means to evaluate the quality of raw materials supplied by suppliers in each time period.

$$\sum_{s=1}^S \widetilde{AVFR}_s \times RQ_{s,t} \leq \widetilde{ACFR} \times \sum_{s=1}^S \text{RM}Q_{s,t} \quad \forall t \quad (10)$$

2. **Supplier capacity:** This indicates the maximum volume of raw material that each supplier can supply in each period.

$$RQ_{s,t} \leq \text{Max}R_{s,t} \quad \forall s, t \quad (11)$$

3. **Supplier service level:** This metric is employed to gauge each supplier's service level, specifically identifying through on-time delivery in each period.

$$\sum_{s=1}^S \overline{AVSL}_s \times RQ_{s,t} \geq \overline{ACSL} \times \sum_{s=1}^S RQ_{s,t} \quad \forall t \quad (12)$$

4. **Raw material availability:** The aggregate raw material resources from all suppliers in each period must encompass the total quantity of required raw materials essential for product manufacturing during that period.

$$RPP \times (RTQ_t + OTQ_t) \leq \sum_{s=1}^S RQ_{s,t} \quad \forall t \quad (13)$$

5. **Raw material inventory:** This denotes the residual stock of raw materials after the manufacturing process in each period.

$$IR_t = IR_{(t-1)} + \sum_{s=1}^S RQ_{s,t} - (RTQ_t + OTQ_t) \times RPP \quad \forall t \quad (14)$$

6. **Product shortages:** This value reveals the number of products in shortfall, representing those that fail to meet customer demand.

$$SPQ_t = SPQ_{(t-1)} + \tilde{D}e_t - PQ_t \quad \forall t \quad (15)$$

7. **Production time availability:** This equation delineates the constraints on production time, encompassing regular and overtime hours, arising from limitations in workforce levels.

$$NW_t \times P \times (RT_t + OT_t) \geq (RTQ_t + OTQ_t) \times PT \quad \forall t \quad (16)$$

8. **Product inventory:** This value presents the residual stock level of products after fulfilling the customer demand in each period.

$$IP_t = IP_{(t-1)} + RTQ_t + OTQ_t - PQ_t \quad \forall t \quad (17)$$

9. **Warehouse space limitation:** This value presents the restricted space within the production facility for raw materials and finished products in each period.

$$(WSP_t \times IP_t) + (WSR_t \times IR_t) \leq MaxWS_t \quad \forall t \quad (18)$$

10. **Workforce balancing:** This equation is utilized to distribute the number of workers in each period equitably.

$$NW_t = NW_{(t-1)} + NHW_t - NFW_t \quad \forall t > 1 \quad (19)$$

11. **Workforce level variation proportion:** This equation is employed to regulate the proportion of variation in workforce levels during each period.

$$NHW_t + NFW_t \leq AWW \times NW_{(t-1)} \quad \forall t \quad (20)$$

12. **Machine capacity:** This value specifies the maximum machine capacity accessible for manufacturing in each period.

$$MU_t \times (RTQ_t + OTQ_t) \leq MaxM_t \quad \forall t \quad (21)$$

13. **Non-negativity constraints:** Eqs. (22) – (25) are established to ensure that the values of all decision variables are non-negative, with some values constrained to be integers.

$$NW_t, NHW_t, NFW_t \geq 0 \text{ and Integer } \forall t \quad (22)$$

$$RTQ_t, OTQ_t, SQ_t, IP_t \geq 0 \text{ and Integer } \forall t \quad (23)$$

$$IR_t, SPQ_t, PQ_t \geq 0 \quad \forall t \quad (24)$$

$$RMQ_{s,t} \geq 0 \quad \forall s, t \quad (25)$$

## 5. Methodology Framework

In this section, we delineate the methodologies of three distinct fuzzy optimization approaches: the traditional weightless fuzzy optimization approach, where no weight is assigned due to DMs preference for equal weight; the fuzzy optimization with proportional fairness approach, where no single objective is unfairly prioritized or favored over other objectives, and the unified proportional fairness and robustness fuzzy optimization approach where all objective functions are equally considered as well as maintained stability under uncertainties. These methodologies present the potential framework and solutions for addressing the APP in SC problems.

Figs. 2-4 demonstrate the procedures of the traditional weightless fuzzy optimization approach, the fuzzy optimization approach with proportional fairness, and the unified fairness and robustness fuzzy optimization approach, respectively. The procedures of these three fuzzy optimization approaches can be divided into four phases: 1. Data Preparation, 2. Defuzzification Process, 3. Membership Function, and 4. Optimization Process. Only the defuzzification process (Phase 2 in the dash block) differentiates these three fuzzy optimization approaches from each other.

### Phase 1: Data Preparation

In this data preparation phase, all parameters will be categorized into crisp and uncertain. Parameters that are precisely known will be grouped into the crisp category, while parameters that are ambiguous and challenging to ascertain will be grouped into the uncertain class. Triangular Fuzzy Number (TFN) is employed here to represent imprecise parameters. According to Zhang et. al. [42], TFN is the most frequently used number in practice that has been applied in many fields such as risk and performance evaluation to represent the uncertainty. It is generally described as three prominent data points of the membership function;  $\tilde{A} = (a^o, a^m, a^p)$  where  $a^o < a^m < a^p$ , represents optimistic, most likely, and pessimistic situations under the triangular distribution, as shown in Fig. 5.

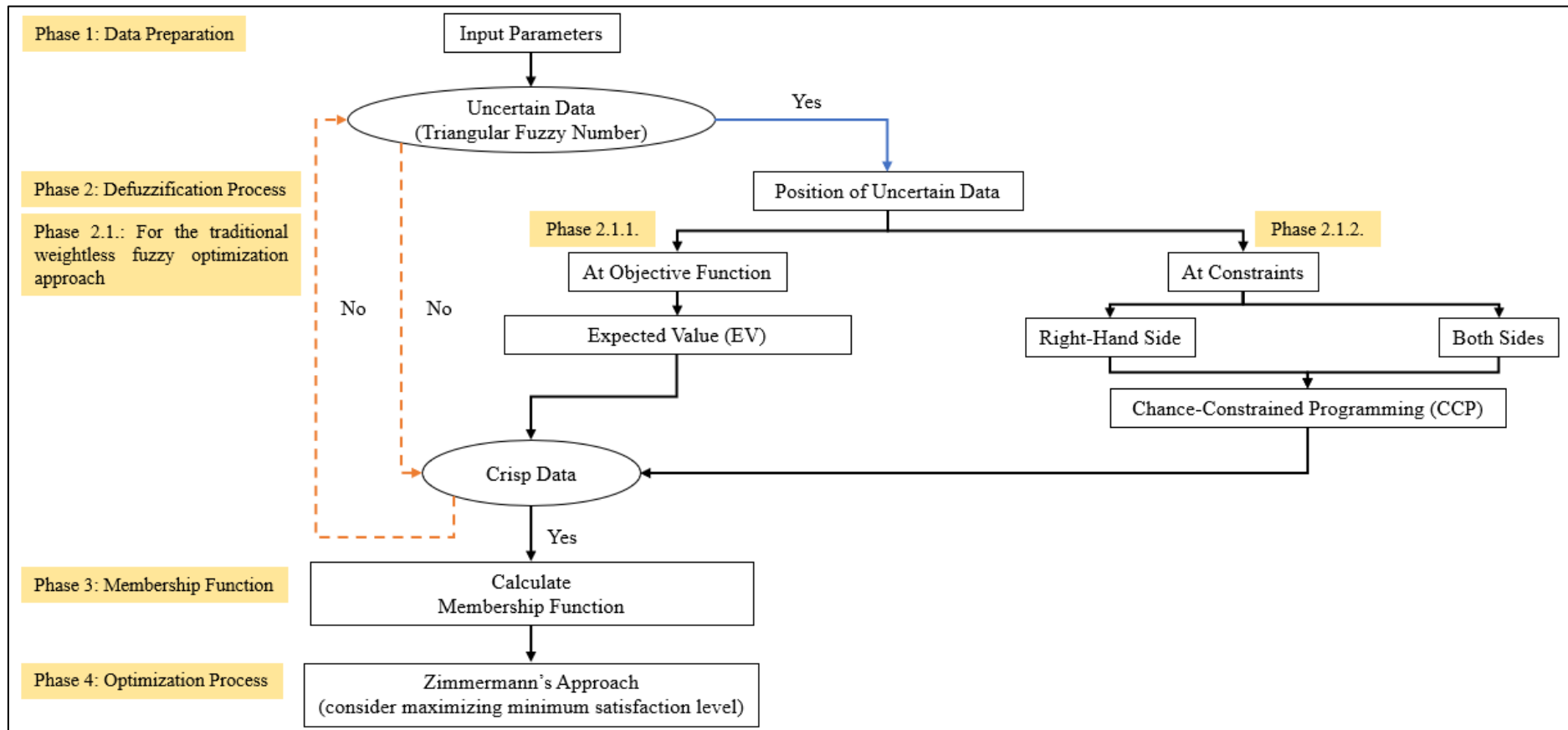


Fig. 2. The procedures of the traditional weightless fuzzy optimization approach.

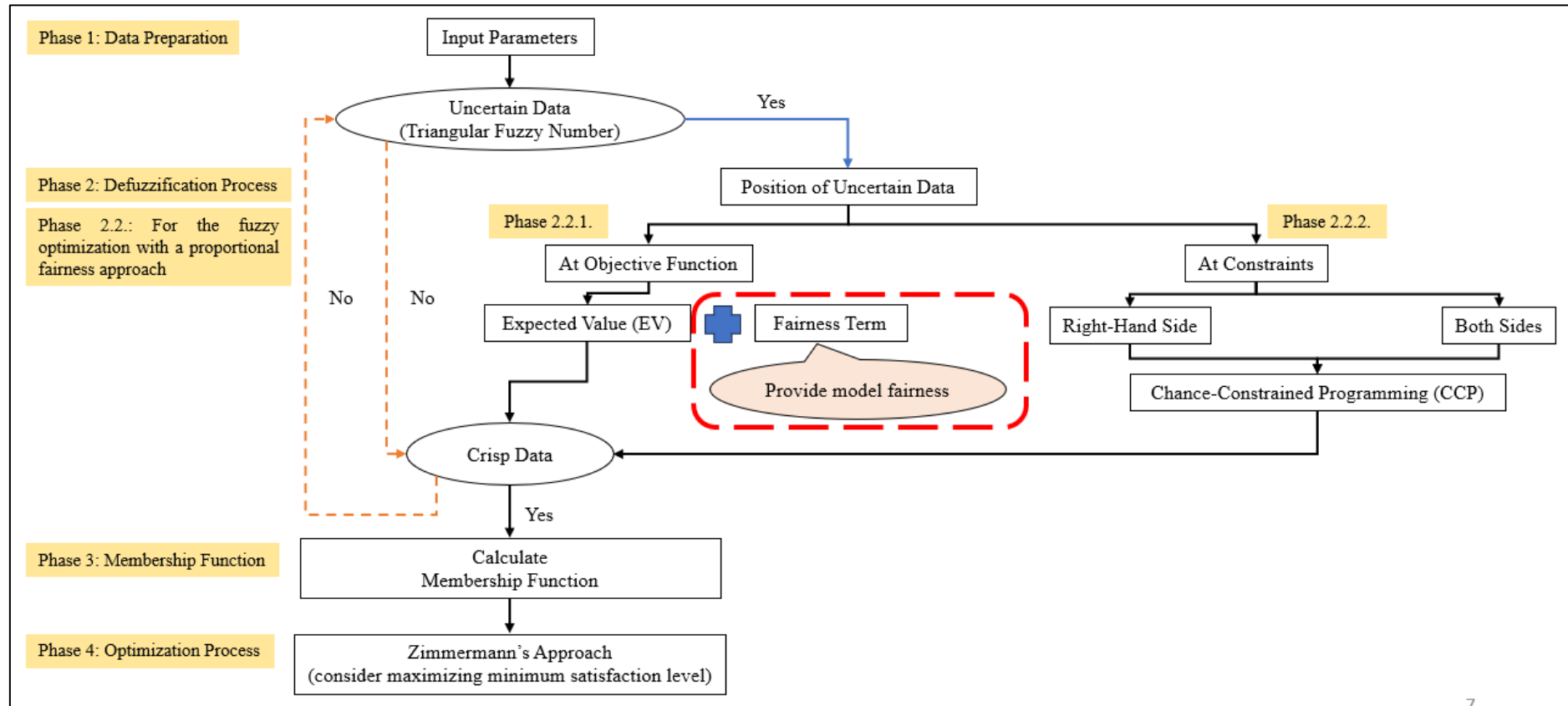


Fig. 3. The procedures of the fuzzy optimization with proportional fairness approach.

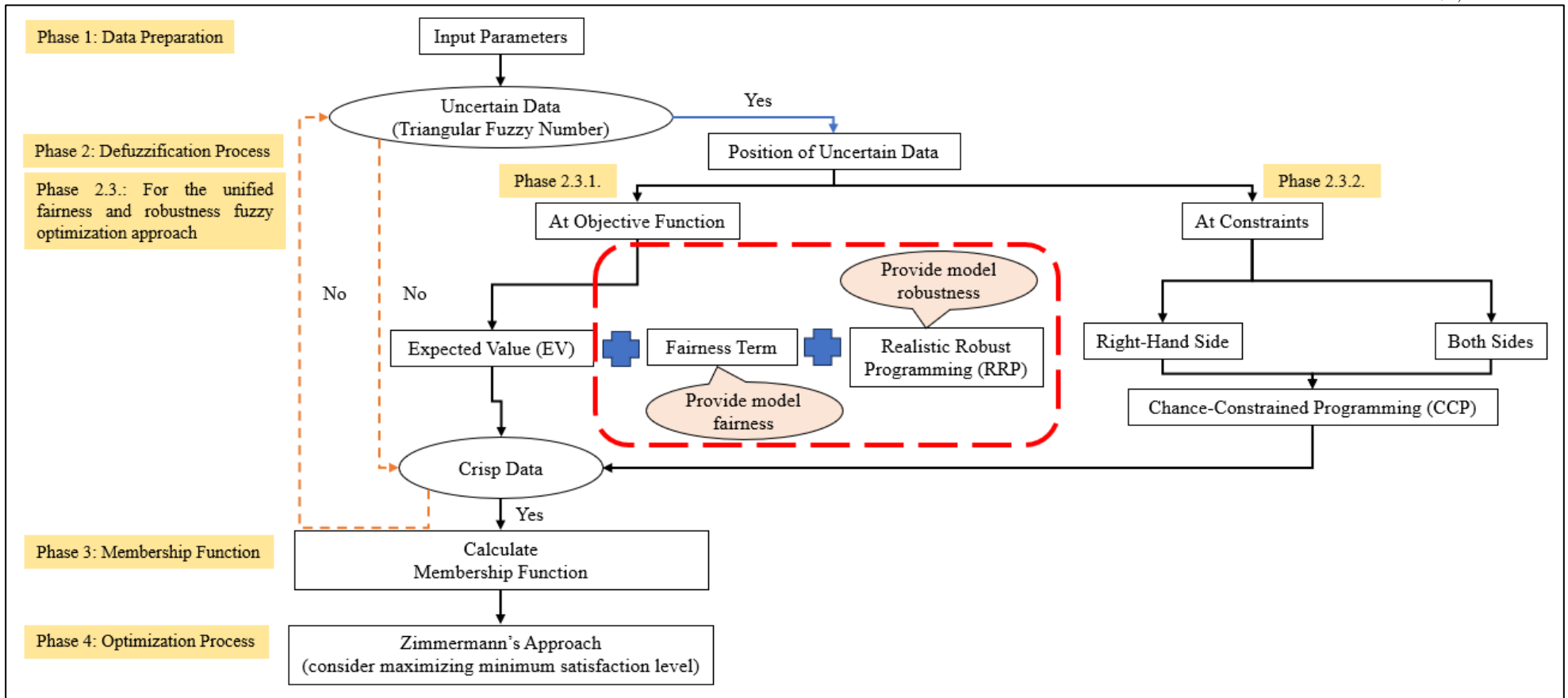


Fig. 4. The procedures of unified fairness and robustness fuzzy optimization approach.

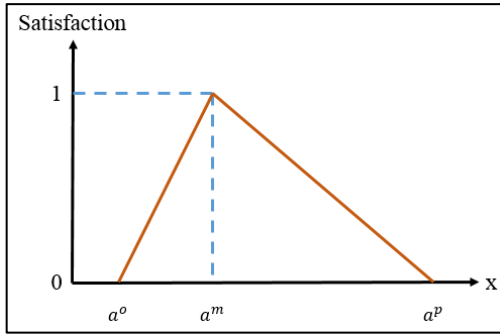


Fig. 5. Triangular Distribution of Triangular Fuzzy Number.

## Phase 2: Defuzzification Process

All uncertain parameters are transformed to crisp parameters in this stage by applying the defuzzification method. The selection of the defuzzification approach is contingent upon the placement of fuzzy parameters within the model, specifically in the objective functions or the constraints.

### Phase 2.1. For the traditional weightless fuzzy optimization approach (as presented in Fig. 3)

#### Phase 2.1.1. Fuzzy data in the objective functions

- Expected Value (EV)

Heilpem [43] first introduced the EV approach as a traditional defuzzification approach for objective function that considers total average performance of the objective function. This defuzzification method has also been applied in many recent works, such as Shen et al. [44], where an EV optimal control model is established for solving an uncertain production inventory problem with deteriorating items.

$$EV = \frac{C_o + (2 * C_m) + C_p}{4} \quad (26)$$

where  $C_o$ ,  $C_m$ , and  $C_p$  are values of the objective coefficient in optimistic, most likely, and pessimistic situations, respectively.

#### Phase 2.1.2. Fuzzy data in the constraints

- Chance-Constrained Programming (CCP)

Chance-Constraint Programming (CCP) is introduced as a fuzzy measurement (credibility) that can be used to convert data fuzziness and simultaneously provide a confident level of constraints. CCP is employed in this study since its function (credibility function ( $\theta$ )) is also related to the robustness of the model. It can be applied with all fuzzy positions in constraints through Eqs. (27)-(28), where they can be applied according to the percentage of credibility ( $\theta$ ). This credibility measures the power of being able to be believed or trustworthy as stated in Li and Liu [45]. The higher the percentage of credibility,

the higher the confidence level that the fuzzy event will occur or the lower the risk of violation. This approach can be applied in several circumstances. For instance, it was recently applied in a study by Huang et al. [46] for solving a decentralized supply chain network problem under the uncertain cost where their results showed that it is appropriate to adopt a chance-constrained approach when the supply chain members can estimate the distributions of the competitor's strategies.

$$Cr\left\{\sum_{j=1}^J a_{ij}x_j \leq \tilde{b}_i\right\} \geq \theta$$

$$(0 \leq \theta \leq 0.5): ax \leq (2\theta)b_m + (1 - 2\theta)b_p \quad (27)$$

$$(0.5 \leq \theta \leq 1): ax \leq (2\theta - 1)b_o + (2 - 2\theta)b_m \quad (28)$$

where  $b_o$ ,  $b_m$ , and  $b_p$  are values of available resources in optimistic, most likely, and pessimistic situations, respectively.  $\theta$  is the percentage of credibility level, which is assigned to 80% in this study.

### Phase 2.2. For the fuzzy optimization with a proportional fairness approach (as presented in Fig. 4)

#### Phase 2.2.1. Fuzzy data in the objective functions

- Expected Value (EV) + Fairness Term

In this process, the fairness term is incorporated into the EV (Expected Value) approach so that the optimal solution of the objective function would not be too far from its Positive Ideal Solution (PIS) in order to enhance the fairness aspect of the model, as outlined below:

#### For minimization objectives:

$$EV = \frac{C_o + (2 * C_m) + C_p}{4} + (Z_i - Z_i^{PIS}) \quad (29)$$

#### For maximization objectives:

$$EV = \frac{C_o + (2 * C_m) + C_p}{4} + (Z_i^{PIS} - Z_i) \quad (30)$$

where  $C_o$ ,  $C_m$ , and  $C_p$  are values of the objective coefficient in optimistic, most likely, and pessimistic situations, respectively.  $Z_i$ ,  $Z_i^{PIS}$ , and  $Z_i^{NIS}$  are values of each objective function, values of the positive ideal solution of each objective function, and values of the negative ideal solution of each objective function, respectively.

#### Phase 2.2.2. Fuzzy data in the constraints

Similar to the previous approach, the Chance-Constrained Programming (CCP) is also employed to defuzzify the fuzzy constraints, as indicated by Eqs. (27)-(28).

**Phase 2.3. For the unified fairness and robustness fuzzy optimization approach (as presented in Fig. 5)**  
**Phase 2.3.1. Fuzzy data in the objective functions**

• **Expected Value (EV) + Fairness Term + Realistic Robust Programming (RRP)**

According to the previous approach, EV only considers the total average performance of the concerned objective function, and the fairness term only considers the equal importance of objective functions where the model robustness has yet to be regarded. Mulvey et al. [47] stated that the model robustness could be identified by optimality robustness and feasibility robustness. The optimality robustness states that the obtained optimal solution of the objective function should be near an ideal optimal solution, whereas the feasibility robustness refers to the case that the obtained solution of all uncertain parameters should be feasible. To allow DMs aware of both total average performance of concerned objective function, consider equally importance of objective functions and model robustness, Robust Programming (RP) approach was developed. Pishvae et al. [48] addressed that RP can be classified into three types: Hard Worst Robust Programming (HWRP), Soft Worst Robust Programming (SWRP), and Realistic Robust Programming (RRP). In this study, the RRP is chosen since it is not only appropriate for profit-seeking and business cases but also generates a reasonable trade-off between optimality and feasibility robustness. As a result, both fairness and RRP (optimality and feasibility terms) are integrated into the EV to augment the fairness and robustness aspects of the model, as delineated in Eqs. (31) – (33).

$$\begin{aligned} \text{RRP} &= \text{Optimality term} + \text{Feasibility term} \\ &= \rho(Z_{max} - Z_{min}) + ((\sigma(d_j^p - (1 - \theta)d_j^m - \theta d_j^p)) \end{aligned} \quad (31)$$

**For minimizing objectives:**

$$\begin{aligned} EV &= \frac{C_o + (2 * C_m) + C_p}{4} + (Z_i - Z_i^{PIS}) \\ &+ \rho(Z_{max} - Z_{min}) + ((\sigma(d_j^p - (1 - \theta)d_j^m - \theta d_j^p)) \end{aligned} \quad (32)$$

**For maximizing objectives:**

$$\begin{aligned} EV &= \frac{C_o + (2 * C_m) + C_p}{4} + (Z_i^{PIS} - Z_i) \\ &+ \rho(Z_{max} - Z_{min}) + ((\sigma(d_j^p - (1 - \theta)d_j^m - \theta d_j^p)) \end{aligned} \quad (33)$$

where  $C_o$ ,  $C_m$ , and  $C_p$  are values of the objective coefficient in optimistic, most likely, and pessimistic situations, respectively.  $Z_i$ ,  $Z_i^{PIS}$ , and  $Z_i^{NIS}$  are values of each objective function  $i$ , values of the positive ideal solution of each objective function  $i$ , and values of the

negative ideal solution of each objective function  $i$ , respectively.  $Z_{max}$  and  $Z_{min}$  are the maximum value and minimum value of the objective function, respectively.  $d_j^o$ ,  $d_j^m$ , and  $d_j^p$  are the optimistic, most likely, and pessimistic values of fuzzy data in each constraint  $j$ , respectively.  $\rho$  is the weight, and  $\sigma$  is the penalty value of a possible violation of each constraint, which are equally assigned to 50% in this study to avoid any bias.  $\theta$  is the percentage of the confidence level, which is assigned to 80% in this study.

**Phase 2.3.2. Fuzzy data in the constraints**

Similar to the previous two approaches, the Chance-Constrained Programming (CCP) is also employed to defuzzify the fuzzy constraints as specified by Eqs. (27)-(28).

**Phase 3: Membership Function**

Multiple objective functions of APP in SC are normal. With multiple objectives, their actual values cannot be mutually evaluated due to the fact that they usually have different units or scales that cannot be directly compared. The membership function is presented as an equation designed to standardize the units of multiple objective functions to a shared scale ranging from 0.0 to 1.0. This standardized scale is referred to as a satisfaction level.

**1. Membership Function for Minimization of the Objective Function**

$$\mu_{z_i} = \begin{cases} 1 & , z_i \leq z_i^{PIS} \\ \frac{z_i^{NIS} - z_i}{z_i^{NIS} - z_i^{PIS}} & , z_i^{PIS} \leq z_i \leq z_i^{NIS} \\ 0 & , z_i \geq z_i^{NIS} \end{cases} \quad (34)$$

**2. Membership Function for Maximization of the Objective Function**

$$\mu_{z_i} = \begin{cases} 1 & , z_i \geq z_i^{PIS} \\ \frac{z_i - z_i^{NIS}}{z_i^{PIS} - z_i^{NIS}} & , z_i^{NIS} \leq z_i \leq z_i^{PIS} \\ 0 & , z_i \leq z_i^{NIS} \end{cases} \quad (35)$$

where  $z_i^{NIS}$  is the maximum bound for minimizing the objective or the minimum bound for maximizing the objective, and  $z_i^{PIS}$  is the maximum bound for maximizing the objective or the minimum bound for minimizing the objective.

## Phase 4: Optimization Process

This phase is applicable for determining the optimal compromise solution in Multi-Objective Fuzzy Linear Programming (MOFLP). Since 1978, Zimmermann's approach [49] has been employed for MOFLP issues as a conventional weightless Fuzzy Linear Programming (FLP) method where the importance of objective functions and constraints are equally considered. The objective is to maximize the minimum value among the satisfaction levels of multiple objective functions, with equal importance assigned to each objective function, as depicted below. In literature, most researchers have used Zimmermann's approach for optimizing MOFLP problems and then set its outcome as a benchmark for comparison. Chanas [50] stated that Zimmermann's approach is a classical FLP that could provide an efficient compromise solution for MOFLP.

$$\begin{aligned} &\text{Maximize } \mu_Z \\ &\text{Subject to: } x \in F(x) \\ &\quad \mu_Z \leq \mu_{Z_i}, \quad i=1, 2, \dots, I \end{aligned} \quad (36)$$

where  $\mu_Z$  is the minimum value of the satisfaction levels from the multiple objective functions and  $\mu_{Z_i}$  is the satisfaction level of each objective function.

## 6. Results and Discussion

The outcomes derived from three distinct fuzzy optimization approaches are presented and compared to assess their efficacy and advantages, focusing on the unified fairness and robustness fuzzy optimization approach. Furthermore, this study also discusses the findings and elucidates the managerial implications stemming from the obtained results.

### 6.1. The outcomes of the traditional weightless fuzzy optimization approach (Zimmermann's approach)

The conventional weightless fuzzy optimization approach is commonly applied to resolve Multiple Objective Fuzzy Linear Programming (MOFLP) where DMs have no preference to prioritize any particular objective, and its optimal solution would be established as a benchmark for comparison, as illustrated in Table 5.

According to Table 5, the traditional weightless fuzzy optimization approach yields the optimal results with the minimum total costs amount of \$129,640, the minimum fluctuations in workforce level of 4 persons, and the maximum total values of purchasing of 1,202 units. The overall satisfaction level is calculated at 39.997%, emphasizing the maximization of the minimum satisfaction level among objective functions. It should be noted that the highest satisfaction level of the objective of maximizing total values of purchasing could cause the problem of fairness as the issue of balancing of stakeholders' objectives is the main concern.

### 6.2. The outcomes of the fuzzy optimization with proportional fairness approach

The optimal solution can be presented in Table 6 after applying the proportional fairness to the model.

According to Table 6, it can be inferred that the fuzzy optimization with proportional fairness approach reveals the minimum total costs amount of \$121,740, the minimum fluctuations in workforce level of 4 persons, and the maximum total values of purchasing of 849 units. The overall satisfaction level is recorded at 42.857%, emphasizing the maximization of the minimum satisfaction level among objective functions.

#### 6.2.1. The comparison between the traditional fuzzy optimization approach and the fuzzy optimization with proportional fairness approach

According to Table 7, the outcomes of the fuzzy optimization with a proportional fairness approach are compared to the results of the traditional weightless fuzzy optimization approach in two aspects (satisfaction level and fairness level).

Table 5. The outcomes of the traditional weightless fuzzy optimization approach.

Objectives	Values	Satisfaction Level
Minimizing Total Supply Chain Costs	\$129,640	39.997%
Minimizing Fluctuation in Workforce Levels	4 persons	42.857%
Maximizing Total Values of Purchasing	1,202 units	85.007%

\*Note that 39.997% is the minimum satisfaction level under maximizing the minimum satisfaction level.

Table 6. The outcomes of the fuzzy optimization with proportional fairness approach.

Objectives	Values	Satisfaction Level
Minimizing Total Supply Chain Costs	\$121,740	49.997%
Minimizing Fluctuation in Workforce Levels	4 persons	42.857%
Maximizing Total Values of Purchasing	849 units	49.975%

\*Note that 42.857% is the minimum satisfaction level under maximizing the minimum satisfaction level.



Table 7. The outcome comparison.

Objectives	Traditional Weightless Fuzzy Optimization Approach			Fuzzy Optimization with Proportional Fairness Approach		
	Objective Values	Satisfaction Level	% Fairness	Objective Values	Satisfaction Level	% Fairness
Minimizing Total Supply Chain Costs	\$129,640	39.997%*	14.398%	\$121,740	49.997%	39.653%
Minimizing Fluctuation in Workforce Levels	4 persons	42.857%	42.857%	4 persons	42.857%*	42.857%
Maximizing Total Values of Purchasing	1,202 units	85.007%	15.053%	849 units	49.975%	40.000%

\*Note that 39.997% and 42.857% are the minimum satisfaction level under maximizing minimum satisfaction level of the Traditional Weightless Fuzzy Optimization Approach and the Fuzzy Optimization Approach with Proportional Fairness, respectively.

## 1. Satisfaction levels and the objective values

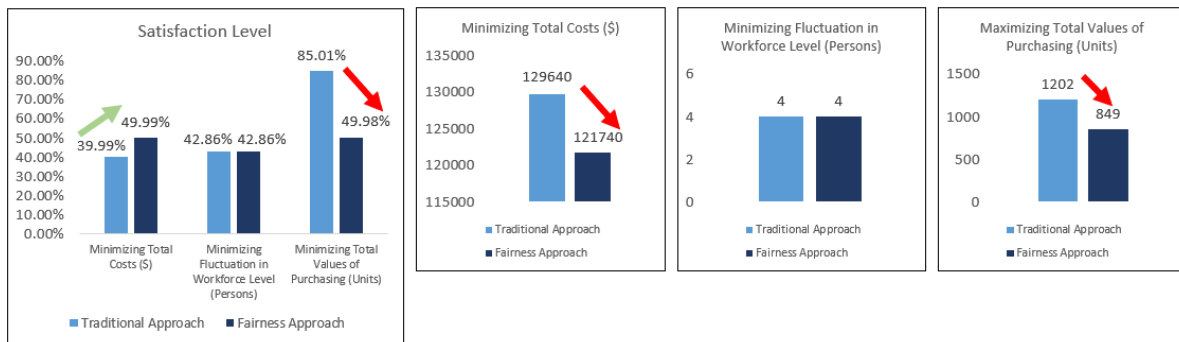


Fig. 6. The satisfaction level and objective value comparison.

According to Fig. 6, it can be seen that having applied the fairness term to the model, the findings reveal that the minimum total costs of the model is decreased from \$129,640 to \$121,740, the minimum fluctuations in workforce level is the same at 4 persons, and the maximum total values of purchasing is reduced from 1,202 units to 849 units. The satisfaction level of minimizing total supply chain costs increases from 39.99% to 49.99%, the satisfaction level of minimizing fluctuations in workforce levels is the same at 42.86%, but the satisfaction level of maximizing total values of purchasing decreases from 85.01% to 49.98%.

## 2. Fairness level

This study employs the proportional fairness to measure the model fairness, where no objective is unfairly prioritized or favored over other objectives. At 0% fairness score, it implies insignificance or neglecting of the

objective, whereas a 100% fairness score indicates the complete prioritization of that objective as the primary focus. This fairness percentage can be calculated as follows:

$$\frac{X_i^{NIS} - X_i}{X_i} \quad (37)$$

where  $X_i$  is the obtained solution of each objective function, and  $X_i^{NIS}$  is the Negative Ideal Solution (NIS) of each objective function.

According to Fig. 7, it can be seen that having applied the fairness term to the model, the outcome reveals that the fairness values of all objective functions are closer to each other than the unbalanced fairness values of the weightless traditional fuzzy optimization approach, representing that the compromised solutions of all objective functions are more fairly and equally considered. Therefore, the model's fairness can be justified.

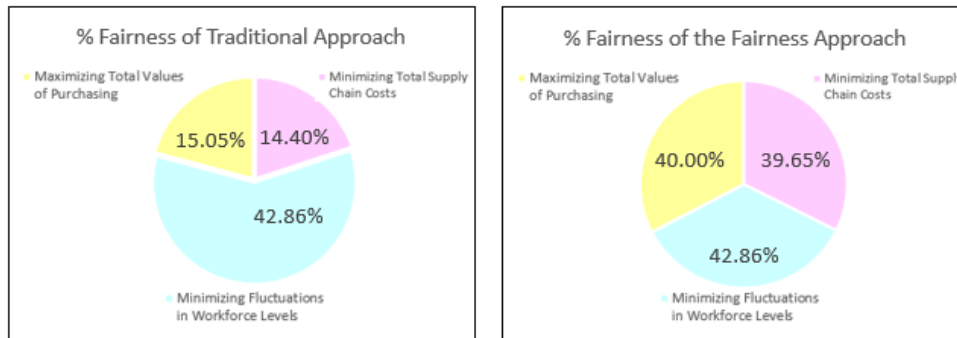


Fig. 7. The fairness level comparison.

**6.3. Outcomes of unified fairness and robustness fuzzy optimization approach**

Having unified both fairness and robustness to the model, the optimal solution can be presented in Table 8.

According to the results in Table 8, it can be concluded that the fairness fuzzy optimization approach yields the minimum total costs amount of \$118,650, the minimum fluctuations in workforce level of 5 persons, and

the maximum total values of purchasing of 768 units. The overall satisfaction level is recorded at 35.143%, emphasizing the maximization of the minimum satisfaction level among objective functions. Specifically, the satisfaction levels for minimizing total supply chain costs, minimizing fluctuations in workforce levels, and maximizing total values of purchasing are 54.868%, 35.143%, and 42.561%, respectively.

Table 8. The outcomes of the unified fairness and robustness fuzzy optimization approach.

Objectives	Values	Satisfaction Level
Minimizing Total Supply Chain Costs	\$118,650	54.868%
Minimizing Fluctuation in Workforce Levels	5 persons	35.143%*
Maximizing Total Values of Purchasing	768 units	42.561%

\*Note that 35.143% is the minimum satisfaction level under maximizing minimum satisfaction level.

Table 9. The outcome comparison.

Objectives	Traditional Weightless Fuzzy Optimization Approach			Fuzzy Optimization with Proportional Fairness Approach			Unified Proportional Fairness and Robustness Fuzzy Optimization Approach		
	Objective Values	Satisfaction Level	% Fairness	Objective Values	Satisfaction Level	% Fairness	Objective Values	Satisfaction Level	% Fairness
Minimizing Total Supply Chain Costs	\$129,640	39.997%	14.398%	\$121,740	49.997%	39.653%	\$118,650	54.868%	42.185%
Minimizing Fluctuation in Workforce Levels	4 persons	42.857%	42.857%	4 persons	42.857%	42.857%	5 persons	35.143%	57.143%
Maximizing Total Values of Purchasing	1,202 units	85.007%	15.053%	849 units	49.975%	40.000%	768 units	42.561%	45.724%

\*Note that 39.997% and 42.857% are the minimum satisfaction level under maximizing minimum satisfaction level for Traditional Weightless Fuzzy Optimization Approach and Fuzzy Optimization Approach with Proportional Fairness, respectively.

6.3.1. The comparison between the weightless traditional fuzzy optimization approach, the fuzzy optimization with proportional fairness approach, and the unified proportional fairness and robustness fuzzy optimization approach.

According to Table 9, the outcomes of the unified fairness and robustness fuzzy optimization approach are compared to the results of the weightless traditional fuzzy

optimization approach and the outcomes of the fuzzy optimization with proportional fairness approach in three aspects as follows:

## 1. Satisfaction levels and objective values

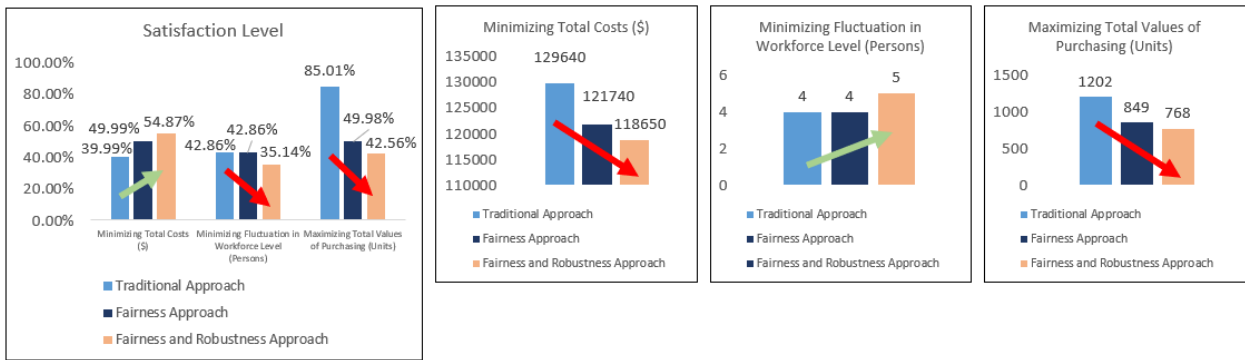


Fig. 8. The satisfaction level and objective value comparison.

According to Fig. 8, it can be concluded that having applied the unified fairness and robustness fuzzy optimization approach, the findings reveal that the minimum total costs are decreased from \$129,640 to \$118,650, the minimum fluctuations in workforce level are increased from 4 persons to 5 persons, and the maximum total values of purchasing is decreased from 1,202 units to

768 units. The satisfaction level of minimizing total supply chain costs increases from 39.99% to 54.87%, the satisfaction level of minimizing fluctuations in workforce levels reduces from 42.86% to 35.14%, and the satisfaction level of maximizing total values of purchasing decreases from 85.01% to 42.56%.

## 2. Fairness level

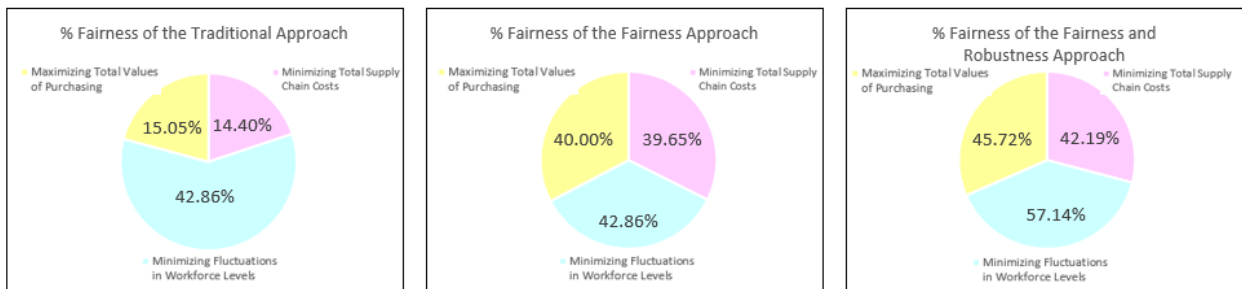


Fig. 9. The fairness levels comparison.

Figure 9 shows the result of integrating both proportional fairness and robustness into the model. It reveals that the percentage of fairness of this unified approach are higher than the percentage of fairness of both traditional weightless fuzzy optimization approach

and fuzzy optimization with proportional fairness approach. In addition, the unified approach makes the percentages of fairness among objective functions still closer to each other.

## 3. Robustness level

To test its ability of model robustness, the results from the unified proportional fairness and robustness fuzzy optimization approach are compared with those from the traditional weightless fuzzy optimization approach and the fuzzy optimization with the proportional fairness approach in terms of average value and standard deviation.

These two measurement techniques are used to evaluate the efficiency and reliability of the optimal solution by experimenting under 10 scenarios, which are subject to the uniform distributed randomly between the pessimistic and optimistic values of fuzzy parameters. Therefore, only a fuzzy objective function (minimization of total supply chain costs) is used to test under these 10 scenarios, as shown in Table 10.

Table 10. The robustness comparison.

Number of Scenario	Traditional Weightless Fuzzy Optimization Approach	Fuzzy Optimization with Proportional Fairness Approach	Unified Proportional Fairness and Robustness Fuzzy Optimization Approach
1	\$99,043.20	\$96,865.60	\$93,688.00
2	\$104,532.28	\$101,749.44	\$97,191.20
3	\$109,297.84	\$106,813.12	\$101,677.60
4	\$114,467.32	\$111,780.16	\$105,266.80
5	\$119,132.88	\$116,043.84	\$109,453.20
6	\$124,378.84	\$121,590.72	\$113,725.60
7	\$129,867.92	\$126,474.56	\$117,428.80
8	\$134,333.48	\$131,038.24	\$121,615.20
9	\$139,802.96	\$136,205.28	\$125,904.40
10	\$144,268.52	\$141,168.96	\$129,990.80
Average	\$121,912.52	\$118,972.99	\$111,594.16
Standard Deviation (SD)	14,459.86	14,113.38	11,632.97
Coefficient of Variation (CV)	0.11861	0.11863	0.10424

According to Table 10, it is observed that the average values of the three fuzzy optimization approaches are closely aligned. Yet, the smallest Coefficient of Variation (CV) of the unified fairness and robustness fuzzy optimization approach can be obtained. This suggests that

the unified fairness and robustness fuzzy optimization approach excels in managing the variability of information, signifying its superiority in controlling the variability of the input data. Consequently, the justification of model robustness is substantiated.

#### 6.4. Sensitivity Analysis

This proposed approach has three parameters: the percentage of credibility ( $\theta$ ),  $\rho$  is the penalty value of a possible violation of objective function, and  $\sigma$  is the penalty value of a possible violation of each constraint, which can be assigned based on DM's preference. These three parameters may affect the outcomes of the interested plans and their comparative results. Therefore, the sensitivity analysis of three parameters will be tested as below.

##### 6.4.1. Sensitivity analysis of the percentage of credibility ( $\theta$ )

As mentioned earlier, the credibility measures the power of being able to believe or be trustworthy which the higher percentage of the credibility, the higher confidence level that the fuzzy event will occur or the lower risk of violation.

According to Table 11, it can be concluded as follows:

- When the percentage of credibility ( $\theta$ ) is varied from 0% to 100%, the minimum total costs is increased from \$94,040 to \$124,080, the minimum fluctuations in workforce level is always the same at 5 persons, and the maximum total values of purchasing is increased from 697 units to 1,001 units.
- In terms of the percentage of satisfaction, the findings reveal that when the percentage of credibility ( $\theta$ ) is higher, presenting low-risk violation, the percentage of satisfaction with minimizing total supply chain costs is lower, the percentage of satisfaction with minimizing fluctuations in workforce levels does not change, and the percentage of satisfaction of maximizing total values of purchasing is lower. This is due to the fact that the higher the satisfaction level, the lower value of the objective with minimization or the higher value of the objective with maximization.
- In terms of the percentage of fairness, the findings reveal that when the percentage of credibility ( $\theta$ ) is higher, presenting low-risk violation, as expected, the percentage of fairness of minimizing total supply chain costs is lower, the percentage of fairness of minimizing fluctuations in workforce levels does not change, the percentage of fairness of maximizing total values of purchasing is also lower. This is due to the fact that the lower percentage of fairness is a result of an increase in the percentage of credibility with lower risk violation.

Table 11. The sensitivity analysis of the percentage of credibility ( $\theta$ ).

% credibility ( $\theta$ )	Objective Values			% Satisfaction of Minimizing Total Supply Chain Costs	% Satisfaction of Minimizing Fluctuations in Workforce Levels	% Satisfaction of Maximizing Total Values of Purchasing	% Fairness of Minimizing Total Supply Chain Costs	% Fairness of Minimizing Fluctuations in Workforce Levels	% Fairness of Maximizing Total Values of Purchasing
	Minimizing Total Supply Chain Cost	Minimizing Fluctuation in Workforce Levels	Maximizing Total Values of Purchasing						
0	\$94,040	5 persons	1,001 units	62.761%	35.143%	50.544%	50.90%	57.14%	53.21%
10	\$97,010	5 persons	989 units	61.653%	35.143%	49.832%	49.82%	57.14%	52.06%
20	\$100,100	5 persons	966 units	60.839%	35.143%	48.456%	48.52%	57.14%	51.68%
30	\$103,190	5 persons	948 units	59.619%	35.143%	47.981%	47.06%	57.14%	50.96%
40	\$106,280	5 persons	905 units	58.922%	35.143%	46.683%	46.31%	57.14%	49.58%
50	\$109,560	5 persons	872 units	57.713%	35.143%	45.167%	45.45%	57.14%	48.81%
60	\$112,450	5 persons	837 units	56.814%	35.143%	44.859%	44.82%	57.14%	47.27%
70	\$115,540	5 persons	794 units	55.547%	35.143%	43.742%	43.39%	57.14%	46.90%
80	<b>\$118,650</b>	<b>5 persons</b>	<b>768 units</b>	<b>54.868%</b>	<b>35.143%</b>	<b>42.561%</b>	<b>42.18%</b>	<b>57.14%</b>	<b>45.72%</b>
90	\$121,710	5 persons	723 units	53.234%	35.143%	41.754%	40.14%	57.14%	44.50%
100	\$124,080	5 persons	697 units	52.146%	35.143%	40.826%	39.72%	57.14%	43.27%

\*Highlighted cell presents the results of applying  $\theta$  at 80% that was used in the case study.

Table 12. The sensitivity analysis of the percentage of credibility ( $\theta$ ) for testing the model robustness.

% credibility ( $\theta$ )	Traditional Weightless Fuzzy Optimization Approach	Fuzzy Optimization with Proportional Fairness Approach	Unified Proportional Fairness and Robustness Fuzzy Optimization Approach
0	\$89,486	\$87,436	\$84,040
10	\$94,221	\$92,932	\$89,010
20	\$99,683	\$97,498	\$93,100
30	\$104,489	\$102,587	\$97,190
40	\$109,464	\$107,420	\$101,280
50	\$114,268	\$112,433	\$105,560
60	\$119,968	\$117,468	\$109,450
70	\$124,438	\$122,373	\$113,540
80	\$129,640	\$127,803	\$118,650
90	\$134,198	\$132,169	\$123,710
100	\$139,549	\$137,249	\$128,080
Average	\$114,491.27	\$112,488.00	\$105,782.73
Standard Deviation (SD)	15,827.08	15,713.25	13,707.42
Coefficient of Variation (CV)	0.13824	0.13969	0.12958

\*Highlighted cell presents the results of applying  $\theta$  at 80% that was used in the case study.

According to Table 12, it can be concluded that when the percentage of credibility ( $\theta$ ) is varied from 0% to 100%, there is no significant effect on the model robustness. The Coefficient of Variation (CV) values (presenting the model robustness) of both traditional weightless fuzzy optimization approach and fuzzy optimization with proportional fairness approach are still close to each other while the CV value of the unified proportional fairness and robustness fuzzy optimization approach is clearly the smallest. Consequently, the model robustness is still be justified.

#### 6.4.2. Sensitivity analysis of the penalty value of a possible violation of objective function ( $\rho$ ) and the penalty value of a possible violation of each constraint ( $\sigma$ )

As mentioned earlier,  $\rho$  is the penalty value of a possible violation of objective function, and  $\sigma$  is the penalty value of a possible violation of each constraint, which are summed to be 1.

Tables 13 and 14 demonstrate the sensitivity analysis of varying the percentages of  $\rho$  and  $\sigma$ , testing the fairness and robustness of the model.

According to Table 13, it can be concluded that when the penalty value of a possible violation of objective function ( $\rho$ ) and the penalty value of a possible violation of each constraint ( $\sigma$ ) are varied, there is no effect on the model robustness. The average values and Standard Deviation (SD) values of all models are still similar to the previous conclusion where the CV value of the unified approach still shows the smallest amount. Consequently, the model robustness still be justified

Table 13. The sensitivity analysis of the percentage of  $\rho$  and  $\sigma$  for testing the model robustness.

$\rho$	$\sigma$	Traditional Weightless Fuzzy Optimization Approach	Fuzzy Optimization with Proportional Fairness Approach	Unified Proportional Fairness and Robustness Fuzzy Optimization Approach
0	100	\$109,764	\$107,788	\$103,296
10	90	\$113,968	\$111,991	\$106,257
20	80	\$117,968	\$115,878	\$109,534
30	70	\$121,836	\$119,767	\$112,171
40	60	\$125,232	\$123,953	\$115,432
50	50	\$129,640	\$127,803	\$118,650
60	40	\$133,187	\$131,029	\$121,843
70	30	\$137,480	\$135,115	\$124,681
80	20	\$141,560	\$139,184	\$127,736
90	10	\$145,050	\$143,191	\$130,914
100	0	\$149,140	\$147,050	\$133,427
Average		\$129,529.55	\$127,522.64	\$118,540.09
Standard Deviation (SD)		12,398.51	12,328.82	9,641.40
Coefficient of Variation (CV)		0.09572	0.09668	0.08133

\*Highlighted cell presents the results of applying  $\rho$  and  $\sigma$  at 50% that was used in the case study

According to Table 14, it can be concluded as follows:

- When the penalty value of a possible violation of objective function ( $\rho$ ) is varied from 0% to 100%, or the penalty value of a possible violation of each constraint ( $\sigma$ ) is varied from 100% to 0%, the minimum total costs is increased from \$103,296 to \$133,427, the minimum fluctuations in workforce level is the same at 5 persons, and the maximum total values of purchasing is decreased from 881 units to 664 units.
- In term of the percentage of satisfaction, the findings reveal that when the penalty value of a possible violation of objective function ( $\rho$ ) is higher, or the penalty value of a possible violation of each constraint ( $\sigma$ ) is lower, the percentage of satisfaction of minimizing total supply chain costs is lower, the percentage of satisfaction of minimizing fluctuations in workforce levels does not change, the percentage of satisfaction of maximizing total values of purchasing is lower. This is due to the fact that the higher the satisfaction, the lower value of the objective with minimization and the higher value of the objective with maximization.
- In terms of the percentage of fairness, the findings reveal that when the penalty value of a possible violation of objective function ( $\rho$ ) is higher, and the penalty value of a possible violation of each constraint ( $\sigma$ ) is conversely lower, the percentage of fairness of minimizing total supply chain costs is lower, the percentage of fairness of minimizing fluctuations in workforce levels does not change, the percentage of fairness of maximizing total values of purchasing is also lower. This is due to the fact increasing the penalty value of a possible violation of the objective function ( $\rho$ ) causes a higher optimality term, which controls the gap between  $Z_{max}$  (maximum value of the objective function) and  $Z_{min}$  (minimum value of the objective function) to be minimized for the model robustness. This forces the obtained result further away from its positive ideal solution. Therefore, the model yields inferior objective values, lowering the percentages of satisfaction level as well as the percentages of fairness.

## 7. Discussion and Managerial Implications

Throughout this study, several noteworthy managerial implications and business insights for decision-makers (DMs) have been identified:

Incorporating multiple objectives of APP in the SC under conditions of uncertainty offers significant advantages and strategic benefits. Pursuing a single objective may prove inadequate in a dynamic business environment marked by unpredictability, diverse challenges, and fluctuating market conditions. By embracing multiple objectives, decision-makers can foster adaptability and resilience in the APP in the SC, effectively navigating uncertainties. For instance, when facing with

supply network disruptions or unexpected demand shifts, diverse objectives allow for a more nuanced decision-making process. A comprehensive APP that accommodates diverse objectives will enable decision-makers to formulate robust risk mitigation plans, addressing vulnerabilities across different aspects of the supply chain. This safeguards against unforeseen challenges and contributes to the long-term stability and sustainability of the entire system.

The incorporation of chance constraint programming to APP in SC also offers numerous benefits. Chance constraint programming introduces a probabilistic element to the traditional deterministic models, enabling a more realistic representation of uncertainties inherent in supply chain processes. By incorporating probabilistic constraints, APP systems can optimize decisions considering the likelihood of different outcomes, leading to reliable plans. This enhances the overall resilience of the supply chain, as the system can adapt to unforeseen events more effectively. Additionally, chance constraint programming enables better risk assessment and mitigation, as decision-makers can analyze the trade-offs between cost and risk. Ultimately, organizations can achieve improved operational performance and responsiveness in the face of uncertainties, enhancing customer satisfaction and competitive advantage.

The incorporation of proportional fairness into the APP in the SC assumes heightened significance in the context of uncertainty. Under conditions of unpredictability, fairness serves as a guiding principle that promotes equitable treatment of stakeholders and fosters resilience within the SC. In an environment where unexpected disruptions and challenges are commonplace, a fair APP helps to mitigate risks by ensuring that decision-making processes consider the diverse interests of stakeholders, including suppliers, manufacturers, distributors, retailers, and customers. Fairness becomes crucial in maintaining positive relationships and collaboration, as it instills a sense of trust among SC members and stakeholders, contributing to a more stable supply chain ecosystem.

- The incorporation of robustness into the APP in the SC is particularly advantageous in the context of uncertainty. In an environment marked by unpredictability and dynamic changes, a robust APP equips organizations with the capacity to maintain stability and operational efficiency. Robustness refers to the system's ability to adapt and perform optimally under varying and uncertain conditions. By building resilience into the APP in the SC, a robust APP ensures that the organization can withstand and recover from disruptions, minimize vulnerabilities, and maintain consistent performance. This adaptability is crucial for navigating uncertainties effectively, enabling the APP in the supply chain to make rapid adjustments and continue operating efficiently in the face of changing circumstances.

Table 14. The sensitivity analysis of  $\rho$  and  $\sigma$ .

$\rho$	$\sigma$	Objective Values			% Satisfaction of Minimizing Total Supply Chain Costs	% Satisfaction of Minimizing Fluctuations in Workforce Levels	% Satisfaction of Maximizing Total Values of Purchasing	% Fairness of Minimizing Total Supply Chain Costs	% Fairness of Minimizing Fluctuations in Workforce Levels	% Fairness of Maximizing Total Values of Purchasing
		Minimizing Total Supply Chain Cost	Minimizing Fluctuation in Workforce Levels	Maximizing Total Values of Purchasing						
0	100	\$103,296	5 persons	881 units	59.87%	35.14%	45.64%	47.54%	57.14%	49.18%
10	90	\$106,257	5 persons	862 units	58.85%	35.14%	45.02%	46.42%	57.14%	48.53%
20	80	\$109,534	5 persons	836 units	57.46%	35.14%	44.85%	45.36%	57.14%	47.25%
30	70	\$112,871	5 persons	811 units	56.55%	35.14%	44.68%	45.07%	57.14%	47.09%
40	60	\$115,432	5 persons	789 units	55.67%	35.14%	43.48%	43.63%	57.14%	46.78%
<b>50</b>	<b>50</b>	<b>\$118,650</b>	<b>5 persons</b>	<b>768 units</b>	<b>54.87%</b>	<b>35.14%</b>	<b>42.56%</b>	<b>42.18%</b>	<b>57.14%</b>	<b>45.72%</b>
60	40	\$121,543	5 persons	747 units	53.46%	35.14%	42.05%	40.72%	57.14%	44.84%
70	30	\$124,681	5 persons	725 units	51.94%	35.14%	41.86%	39.16%	57.14%	44.55%
80	20	\$127,436	5 persons	701 units	50.51%	35.14%	41.23%	38.22%	57.14%	43.31%
90	10	\$130,214	5 persons	683 units	49.47%	35.14%	40.75%	37.34%	57.14%	43.12%
100	0	\$133,427	5 persons	664 units	48.36%	35.14%	40.43%	36.19%	57.14%	42.85%

\*Highlighted cell presents the results of applying  $\rho$  and  $\sigma$  at 50% that was used in the case study



Furthermore, a robust APP contributes to risk management by actively identifying potential vulnerabilities and proactively addressing them. Rather than being reactive to disruptions, organizations with a robust supply chain system are better positioned to anticipate, plan for, and mitigate risks before they escalate. This forward-looking approach minimizes the impact of uncertainties and contributes to the overall stability of the supply chain. In addition, a robust APP supports efficient resource allocation by optimizing decision-making processes. It ensures that resources, such as inventory, transportation, and production capacity, are allocated effectively to maintain operational continuity and meet fluctuating demands under uncertain conditions.

These insights emphasize the importance of adopting an unified fairness and robustness fuzzy optimization approach for creating resilient and equitable long-term APP plans in SC. The proposed framework is particularly valuable for decision-makers aiming to navigate the complexities of real-world supply chain management.

## 8. Conclusions

The study conclusively established the superiority of the proposed unified fairness and robustness fuzzy optimization approach over the traditional fuzzy optimization approach. Through a comprehensive exploration of a multiple-objective APP in the SC problem, the study demonstrated that the proposed framework with the unified fairness and robustness fuzzy optimization approach achieved heightened levels of fairness and robustness, which are crucial for effective supply chain planning.

A key takeaway from this study is the effective handling of uncertainties in supply chain dynamics. By simultaneously and equally optimizing various conflicting objectives where the minimization of total supply chain costs, the minimization fluctuations in workforce levels, and the maximization of total values of purchasing under uncertainty of costs, customer demands, suppliers' service level, and failure rate of raw materials, the proposed approach showcases its versatility and applicability in real-world scenarios where uncertainties are prevalent.

This unified approach extended the capabilities of traditional fuzzy optimization methodologies by incorporating innovative elements. The utilization of Triangular Fuzzy Numbers (TFN) for representing imprecise data and introducing a Fairness Term and Realistic Robust Programming (RRP) significantly contributed to improved fairness and robustness in the optimization process. In addition, chance constraint programming enabled better risk assessment and mitigation, as decision-makers can analyze the trade-offs between cost and risk. Ultimately, organizations can achieve improved operational performance and responsiveness in the face of uncertainties, enhancing customer satisfaction and competitive advantage.

The obtained optimal solutions highlighted the efficacy of the proposed approach, particularly in

scenarios characterized by high levels of conflict among multiple objective functions. The proposed framework demonstrated a remarkable capacity for resolving intricate and conflicting optimization challenges, making it well-suited for addressing the complexities inherent in APP plans in SC.

While the study presented valuable insights, it also identified certain limitations, such as the lack of restrictions on the degree of fuzziness and the potential exploration of alternative distributions for fuzzy parameters. This calls for further research to refine and expand the model's capabilities, potentially incorporating advanced meta-heuristic algorithms in more complex scenarios for enhanced optimization outcomes.

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